Forms and formulas for linear equations

*General form of a line*: \( Ax + By = C \), where \( A, B, \) and \( C \) are constants, i.e. any real numbers.

*Midpoint formula*: \( M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \), where \( (x_1, y_1) \) & \( (x_2, y_2) \) are any two points on the line.

*Slope formula*: \( m = \frac{y_2 - y_1}{x_2 - x_1} \), where \( (x_1, y_1) \) & \( (x_2, y_2) \) are any two points on the line.

*Slope-intercept form*: \( y = mx + b \), \( m \) is the slope and \( (0,b) \) is the \( y \)-intercept.

*Point-slope form*: \( y - y_1 = m(x - x_1) \), \( m \) is the slope and \( (x_1, y_1) \) is a point on the line.

*Distance formula*: \( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \), where \( (x_1, y_1) \) & \( (x_2, y_2) \) are two points on the line.

*Pythagorean formula*: \( a^2 + b^2 = c^2 \), where \( a \) & \( b \) are the sides of a right triangle and \( c \) is the hypotenuse.

Some facts about linear equations

*Positive slope*: If \( m > 0 \), then the line falls to the left and rises to the right.

*Negative slope*: If \( m < 0 \), then the line falls to the right and rises to the left.

*Vertical Lines*: If a line is vertical, its slope is undefined and it is of the form \( x = c \), where \( c \) is a constant.

*Horizontal Lines*: If a line is horizontal, its slope is 0 and it is of the form \( y = c \), where \( c \) is a constant.

*Parallel lines*: If two lines are parallel \( (l_1 \parallel l_2) \), then their slopes are equal, i.e. \( m_1 = m_2 \).

*Perpendicular lines*: If two lines are perpendicular \( (l_1 \perp l_2) \), then their slopes are the negative reciprocals of one another, i.e. \( m_1 = -\frac{1}{m_2} \) or equivalently \( m_1 \cdot m_2 = -1 \).

Some facts about systems of linear equations (two equations)

*Lines intersect at one point*: Slopes are different, the system is consistent and the lines are independent since there is precisely one solution, which is a point on the coordinate plane.

*Lines are parallel (do not intersect)*: Slopes are the same, \( y \)-intercepts are different, the system is inconsistent and the lines are independent since there is no solution, which we can denote as \( \emptyset \).

*Lines are the same*: Slopes and \( y \)-intercepts are the same, the system is consistent and the lines are dependent since there are an infinite number of solutions, i.e. all real numbers, which we denote as \( (-\infty, \infty) \).

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