Module 4 – Conversion Factors

<u>Prerequisite</u>

Module 4 *requires* Lessons 1A and 1B (on exponential fundamentals) and 2A and 2B (on metric fundamentals). Lesson 2C and Module 3 will be helpful, but not essential, for *most* of the problems in Module 4.

If you are in a hurry to catch up in order to begin conversion factors, you may return to the other lessons in Modules 1-3 at a later time. (The other lessons in Modules 1-3 will be needed for later modules.)

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Introduction

The material in Module 4 may be a review of what you have learned previously. Each lesson will include suggestions for how you can complete this review quickly.

However, some rules in Module 4 that you may not know, such as "writing the wanted unit first," and using the "starting template." These rules will be important in solving complex problems in later lessons. Be sure to read each section and do at least the last two problems in each practice set.

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Lesson 4A: Conversion Factor Basics

Conversion factors can be used to change from one unit of measure to another, or to find measures of substances or processes that are equivalent.

A conversion factors is a *fraction* that equals *one*. Multiplying a quantity by a conversion factor changes the *units* that measure a quantity but does not change the original amount of the quantity.

Conversion factors equal unity (1) because they are made from equalities. For any fraction in which the top and bottom are equal, its value is *one*.

For example: $\frac{7}{7} = 1$ Or, since 1000 milliliters = 1 liter; $\frac{1000 \text{ mL}}{1 \text{ liter}} = 1$, and $\frac{1 \text{ liter}}{1000 \text{ mL}} = 1$

These last two fractions are typical conversion factors. Any fraction that equals *one* rightside up will also equal *one* up-side down. Any conversion factor can be inverted (flipped over) for use if necessary, and it will still be equal to one.

"Conversion factor" is a term for a ratio, or fraction, or two measured quantities that are equal or equivalent in a problem. Our method of solving problems will focus on finding equal or equivalent quantities. We will use those equalities to construct conversion factors.

Let's try an example of conversion-factor math. Try the following problem. Show your work on the page or in your problem notebook, then check your answer below.

<u>Multiply</u> 0.75 kilometers • <u>1000 meters</u> = 1 kilometer

* * * * * (* * * mean: cover below, write your answer, *then* check below.)

Answer

 $0.75 \text{ kilometers} \bullet \underline{1000 \text{ meters}} = \underline{(0.75 \bullet 1000)} \text{ meters} = 750 \text{ meters}$ $1 \text{ kilometer} \qquad 1$

When these terms are multiplied, the "like units" on the top and bottom cancel, leaving meters as the unit on top.

Since the conversion factor multiplies the *given* quantity by one, the answer equals the *given* amount that we started with. This answer means that 750 meters is the same as 0.75 km.

Multiplying by conversion factors changed the unit, but not the amount, of the *given* quantity. The result is what we started with, measured in different units.

This process answers a question posed in many science problems: From the units we are given, how can we obtain the units we want?

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Summary

- Conversion factors are made from ratios, fractions, or two measured quantities that are equivalent, related, or equal in a problem.
- Conversion factors equal one, because the top and bottom terms are equal or equivalent.
- When the units cancel correctly and the numbers are where they should be to make legal conversion factors, the answer will be a correct conversion from the starting *given* units to the final *wanted* units.
- → *Units* tell you where to place the numbers to solve a calculation correctly.

Practice: Try every other lettered problem. Check your answers frequently. If you miss one on a section, try a few more. Answers are on the next page.

1. Multiply the conversion factors. Cancel units that cancel, then group the numbers and do the math. Write the answer number and unit.

2. To be legal, the top and bottom of conversion factors must be equal. Label these conversion factors as *legal* or *illegal*.

a. <u>1000 mL</u>	b. <u>1000 liters</u>	с. <u>1.00 g H</u> 2O	d.	1 volt
1 liter	1 milliliter	1 mL H ₂ O	1	100 centivolts

- e. 1 mL f. 10^3 cm^3 g. 1000 kilowatts h. 1 kilocalorie1 cc 1 L 1 watt 10^3 calories
- 3. Add numbers to make legal conversion factors, with at least one of the numbers in each conversion factor being a 1.

a.	<u> </u>	b	<u>mole</u> nanomole	с.	<u> </u>
d.	centijoules	e	liters	f.	curie
	joules		cubic cm		picocurie

4. Finish these.

a.
$$27A \cdot \underline{2T}_{8A} \cdot \underline{4W}_{3T} =$$

b. 2.5 meters $\cdot \underline{10^2 \text{ cm}}_{1 \text{ meter}} =$

- c. 33 grams $\frac{1 \text{ kilogram}}{10^3 \text{ grams}}$ =
- d. $\underline{95 \text{ km}}$ $\underline{0.625 \text{ miles}}$ = hour 1 km
- e. $\underline{27 \text{ meters}} \bullet \underline{60 \text{ s}} \bullet \underline{60 \text{ min.}} \bullet \underline{1 \text{ kilometer}} =$ seconds $1 \text{ min.} 1 \text{ hour} \quad 10^3 \text{ meters}$

ANSWERS

1	a.	225 centigrams • <u>1 gram</u> • <u>1 kilogram</u> = <u>225 x 1 x 1</u> kg = 0.00225 kilograms
		100 centigrams 1000 grams 100 x 1000 or 2.25 x 10⁻³ kg
		The answer means that 0.00225 kg is equal to 225 cg.
	b.	1.5 hours • <u>60 minutes</u> • <u>60 seconds</u> = <u>1.5 x 60 x 60</u> s = 5,400 s or 5.4 x 10^3 s 1 hour 1 minute 1
		Recall that s is the SI abbreviation for seconds. This answer means that 1.5 hours is equal to 5,400 seconds.
2.	a.	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
		Legal Illegal Legal IF <i>liquid</i> water Legal
	e.	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
3.	a.	1000 gramsor1 gramb.1 moleor10 ⁻⁹ mole1 kilogram 10^{-3} kilogram 10^9 nanomole1 nanomole
	c.	$\begin{array}{c c} 1 \text{ cm}^3 \\ 1 \text{ mL} \end{array} \begin{array}{c} \text{d. } \underline{100 \text{ centijoules}} & or & \underline{1 \text{ centijoule}} \\ 1 \text{ joule} & 1 \text{ joules} \end{array}$
	e.	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
4.	a.	$27A \cdot \underline{2T}_{8A} \cdot \underline{4W}_{3T} = 27A \cdot \underline{2\mp}_{8A} \cdot \underline{4W}_{3\mp} = \underline{27 \cdot 2 \cdot 4}_{8 \cdot 3} \cdot W = 9W$
	b.	2.5 meters • $10^2 \text{ cm} = 2.5 \text{ x} 10^2 = 250 \text{ cm}$ 1 meter
	c.	$33 \frac{\text{grams}}{10^3 \frac{1}{\text{grams}}} = \frac{33}{10^3} = 0.033 \text{ kg}$
	d.	$\frac{95 \text{ km}}{\text{hour}} \cdot \frac{0.625 \text{ miles}}{1 \text{ km}} = \frac{95 \cdot 0.625}{1} = 59 \text{ miles}$
	e.	$\frac{27 \text{ meters}}{\text{seconds}} \bullet \frac{60 \text{ s}}{1 \text{ min}} \bullet \frac{60 \text{ min}}{1 \text{ hour}} \bullet \frac{1 \text{ kilometer}}{10^3 \text{ meters}} = \frac{27 \cdot 60 \cdot 60}{10^3} = 97 \text{ km}$
*	* *	* *

Lesson 4B: Single Step Conversions

In the previous lesson, conversion factors were supplied. In this lesson, you will learn to make your own conversion factors to solve problems.

Let's use this simple word problem as an example.

Q. How many years is 925 days?

In your notebook, write an answer to each step below.

Steps for Solving with Conversion Factors

- 1. Begin by writing a question mark (?) and then the *unit* you are looking for in the problem, the *answer unit*.
- 2. Next write an equal sign. It means, "OK, that part of the problem is done. From here on, leave the answer unit alone." You don't cancel the answer unit, and you don't multiply by it.
- 3. After the = sign, write the number and unit you are *given* (the known quantity).

At this point, in your notebook should be ? years = 925 days

- 4. Next, write a and a line _____ for a conversion factor to multiply by.
- 5. A key step: write the *unit* of the *given* quantity in the denominator (on the bottom) of the conversion factor. Leave room for a number in front.

Do not put the given number in the conversion – just the given unit.

? years = 925 days • _____

days

6. Next, write the answer unit on the top of the conversion factor.

? years = 925 days • <u>year</u> days

- 7. Add numbers that make the numerator and denominator of the conversion factor *equal*. In a legal conversion factor, the top and bottom quantities must be equal or equivalent.
- 8. Cancel the units that you set up to cancel.
- 9. *If* the unit on the right side after cancellation is the answer unit, stop adding conversions. Write an = sign. Multiply the *given* quantity by the conversion factor. Write the number and the un-canceled unit. Done!

Finish the above steps, then check your answer below.

* * * * * ? years = 925 days • <u>1 year</u> = <u>925 years</u> = **2.53 years 365** days 365

(Sig figs: 1 is exact, 925 has 3 sf, 365 has 3 sf (1 yr. = 365.24 days), round to 3 sf.)

You may need to look back at the above steps on occasion, but you should not need to memorize them. By doing the following problems, you will quickly learn what you need to know.

* * * * *

Practice: After each numbered problem, check your answers at the end of this lesson. Look back at the steps if needed.

In the problems in this practice section, write conversions in which *one* of the numbers (in the numerator *or* the denominator) is a 1.

If these are easy, do every second or third letter. If you miss a few, do a few more.

- 1. Add numbers to make these conversion factors legal, cancel the units that cancel, multiply the *given* by the conversion, and write your answer.
 - a. ? cm = 0.35 meters <u>cm</u> = 1 meter
 b. ? days = 96 hours <u>day</u> = 24 hours
 c. ? mL = 3.50 liters <u>mL</u> =
 - d. ? minutes = 330 s _____ =
 - seconds
- 2. To start these, put the *unit* of the *given* quantity where it will cancel. Then finish the conversion factor, do the math, and write your answer with its unit.
 - a. ? seconds = 0.25 minutes $\underline{sec.} = 1$ b. ? kilograms = 250 grams • $\underline{kilogram} = 10^3$ c. ? meters = 14 cm • $\underline{cm} = cm$ d. ? days = 2.73 years • $\underline{365} = 10^3$
 - e. ? years = 200. days <u>1</u> =

3. You should not need to memorize the written rules for arranging conversion factors; the steps will quickly become automatic with practice. However, it is helpful to memorize this "single unit starting template."



The template emphasizes that your first conversion factor puts the given *unit* (but *not* the given *number*) where it will cancel.

=

- a. ? months = 5.0 years _____ =
- b. ? liters = 350 milliliters =
- c. ? minutes = 5.5 hours
- 4. Use the starting template to find how many hours equal 390 minutes.
 - ?
- 5. ? milligrams = 0.85 kg _____ gram _____ = kg gram

* * * * *

AN\$WER\$

Some but not all unit cancellations are shown. For your answer to be correct, it must include its unit.

Your conversions may be in different formats, such as 1 kg = 1,000 g or $1 \text{ g} = 10^{-3} \text{ kg}$, as long as the top and bottom are equal and you get the same answers as below.

1. a. ? cm = $0.35 \text{ meters} \cdot \frac{100 \text{ cm}}{1 \text{ meter}} = 0.35 \cdot 100 = 35 \text{ cm}$

(Sig figs: 0.35 has 2 sf, prefix definitions are exact with infinite sf, answer is rounded to 2 sf)

b. ? days = 96 hours • $1 \text{ day}_{24 \text{ hours}}$ = $\frac{96}{24}$ = 4.0 days

c. ? mL =
$$3.50 \frac{\text{liters}}{10^3 \text{ mL}} = 3.50 \cdot 10^3 = 3.50 \times 10^3 \text{ mL}$$

1 $\frac{10^3 \text{ mL}}{1 \text{ liter}}$

(Sig figs: 3.50 has 3 sf, prefix definitions are exact with infinite sf, answer is rounded to 3 sf)

d ? minutes =
$$330 \frac{\text{sec.}}{60} \cdot \frac{1 \text{ minute}}{60} = \frac{330}{60} = 5.5 \text{ minutes}$$

(Sig figs: 330 has 2 sf, this time definition is exact with infinite sf, answer is rounded to 2 sf)

2. a.
$$? \operatorname{seconds} = 0.25 \operatorname{minutes} \cdot \underbrace{60 \operatorname{sec.}}_{1 \operatorname{minute}} = 0.25 \cdot 60 = 15 \operatorname{sec.}$$

b. $? \operatorname{kilograms} = 250 \operatorname{grams} \cdot \underbrace{1 \operatorname{kilogram}}_{10^3} = \underbrace{250}_{10^3} = 0.25 \operatorname{kg}}_{10^3} \operatorname{grams} = 10^3 \operatorname{grams} = 10.54 \operatorname{gram} = 10^3 \operatorname{grams} = 10$

Lesson 4C: Multi-Step Conversions

In Problem 5 at the end of the previous lesson, we did not know a direct conversion from kilograms to milligrams. However, we knew a conversion from kilograms to grams, and another from grams to milligrams.

In most problems, you will not know a single conversion from the *given* to wanted unit, but there will be known conversions that you can *chain together* to solve.

Solving With Multiple Conversions

If the unit on the right after you cancel units is *not* the answer unit, get rid of it. Write it in the next conversion factor where it will cancel.

Finish the next conversion with a known conversion, one that either *includes* the answer unit, or gets you *closer* to the answer unit.

In making conversions, set up *units* to *cancel*, but add *numbers* that make *legal* conversions.

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Practice

Write the six metric *fundamentals* from memory. Use those fundamentals for the problems below.

These are in pairs. If Part A is easy, go to Part A of the next question. If you need help with Part A, do Part B for more practice.

ANSWERS

For visibility, not all cancellations are shown, but cancellations should be marked on your paper.

1a. ? kilograms = 760 milligrams •
$$1 g$$
 • $1 kg$ = 7.6 x 10⁻⁴ kg
10³ mg 10³ g
b. ? cg = 4.2 kg • $10^3 g$ • $10^2 cg$ = 4.2 x 10⁵ cg
1 kg 1 g

2a. ? years =
$$2.63 \times 10^4$$
 hours • $1 \frac{day}{24 hr.}$ • $1 \frac{yr}{365 days}$ = 2.63×10^4 = 3.00 years
b. ? seconds = 1.00 days • $24 \frac{hr}{1 day}$ • $\frac{60 \text{ min}}{1 \text{ hr}}$ • $\frac{60 \text{ sec}}{1 \text{ min}}$ = 8.64×10^4 sec.
3a. ? mg H₂O₍₁₎ = $1.5 \text{ cc} \text{ H}_2\text{O}_{(1)}$ • $\frac{1 \text{ g} \text{ H}_2\text{O}_{(1)}}{1 \text{ cc} \text{ H}_2\text{O}_{(1)}}$ • $\frac{103 \text{ mg}}{1 \text{ g}}$ = $1.5 \times 10^3 \text{ mg} \text{ H}_2\text{O}_{(1)}$
b. ? kg H₂O₍₁₎ = $5.5 \text{ liter} \text{ H}_2\text{O}_{(1)}$ • $\frac{1000 \text{ mL}}{1 \text{ L}}$ • $\frac{1.00 \text{ g} \text{ H}_2\text{O}_{(1)}}{1 \text{ mL} \text{ H}_2\text{O}_{(1)}}$ • $\frac{1 \text{ kg}}{40^3 \text{ g}}$ = $5.5 \text{ kg} \text{ H}_2\text{O}_{(1)}$
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Lesson 4D: English/Metric Conversions

Using Familiar Conversions

All of the unit conversions between units that we have used so far have had the number **1** on either the top or the bottom, but a one is not required in a legal conversion.

Both "1 kilometer = 1,000 meters" and "3 kilometers = 3,000 meters" are true equalities, and both equalities could be used to make legal conversion factors. In most cases, however, conversions with a 1 are preferred.

Why? We want conversions to be *familiar* and easy to remember, so that we can write them automatically, and quickly check that they are correct. Definitions are usually based on *one* of a unit, such as "1 km = 1,000 meters." Definitions are the most frequently encountered conversions, and are therefore the most familiar.

Other conversions may be familiar even if they do not include a **1**. For example, many cans of soft drinks are labeled "12.0 fluid ounces (355 mL)." This supplies an equality for English-to-metric volume units: 12.0 fluid ounces = 355 mL. That is a legal conversion and, because its numbers and units are seen often, it is a good conversion to use because it is easy to remember.

Bridge Conversions

Science problems often involve a key *bridge* conversion between one unit system, or one substance, and another.

For example, a *bridge* conversion between the metric and English-system *distance* units is

2.54 centimeters = 1 inch

Using this equality, we can convert back and forth between metric and English measurements of distance.

Any metric-English distance equality can be used to convert between distance measurements in the two systems. 1 mile = 1.61 km is another metric-English conversion for distance that is frequently used. However, in our initial practice in these lessons, the *cm-to-inch* conversion will be the one used most often.

In problems that require bridge conversions, our strategy to begin will be to "head for the bridge," to *begin* by converting to one of the two units in the *bridge* conversion.

When a problem needs a bridge conversion, use these steps.

- 1) First convert the *given* unit to the one of the two *bridge units* that is in the *same system* as the *given* quantity.
- 2) Next, multiply by the bridge conversion. The bridge conversion crosses over from the *given* system ito the WANTED system of the *answer*.
- 3) Multiply by other conversions in the WANTED system to get the final answer *unit* WANTED.

Conversions between the metric and English measurement systems provide an way to practice the bridge-conversion methods we will use in chemical reaction calculations.

Add these English distance-unit definitions to your list of memorized conversions.

12 inches = 1 foot 3 feet = 1 yard 5,280 feet = 1 mile

Memorize and use this metric to English bridge conversion for distance.

2.54 centimeters = 1 inch

Then cover the answer below and apply the steps and conversions above to this problem.

Q. ? feet = 1.00 meter

* * * * *

Answer

Since the *wanted* unit is English, and the *given* unit is metric, an English/metric bridge is needed.

<u>Step 1</u>: Head for the bridge. Since the *given* unit (meters) is metric system, convert to the *metric unit* used in the bridge conversion (2.54 cm = 1 inch) *-- centimeters*.

? feet = 1.00 meter • $\frac{100 \text{ cm}}{1 \text{ meter}}$ • ____

Note the start of the next conversion. Since cm is not the wanted answer unit, cm *must* be put in the next conversion where it will cancel. If you *start* the "next unit to cancel" conversion automatically after finishing the prior conversion, it helps to arrange and choose the next conversion.

Adjust and complete your work if needed.

* * * * *

Step 2: Complete the bridge that converts to the *system* of the answer: English units.

? feet = $1.00 \text{ meter} \cdot \underline{100 \text{ cm}} \cdot \underline{1 \text{ inch}} \cdot \underline{1 \text{ inch}}$ inch

Step 3: Get rid of the unit you've got. Get the unit you want.

? feet = $1.00 \text{ meter} \cdot 100 \text{ cm} \cdot 1 \text{ inch} \cdot 1 \text{ foot} = 3.28 \text{ feet}$ 1 meter 2.54 cm 12 inches

The answer tells us that 1.00 meter (the *given* quantity) is equal to 3.28 feet.

Some science problems take 10 or more conversions to solve. However, if you know that a bridge conversion is needed, "heading for the bridge" breaks the problem into pieces. Solving in pieces will simplify your navigation to the answer.

* * * * *

Practice: Use the bridge conversion above. Start by doing every other problem. Do more if you need more practice.

1. ? cm = 12.0 inches • ____ =

- 2. ? inches = 1.00 meters _____ _____
- 3. For ? inches = 760. mm
 - a. To what unit to you aim to convert the *given* in the initial conversions? Why?
 - b. Solve: ? inches = 760. mm
- 4. ? mm = 0.500 yards
- 5. For ? km = 1.00 mile, to convert using 1 inch = 2.54 cm,
 - a. To what unit to you aim to convert the *given* in the *initial* conversions? Why?
 - b. Solve: ? km = 1.00 mile
- 6. Use as a bridge conversion for mass units, 1 kilogram = 2.2 lbs.

? grams = 7.7 lbs

7. Use the "soda can" volume conversion (12.0 fluid ounces = 355 mL).

? fluid ounces = 2.00 liters

ANSWERS

In these answers, some but not all of the unit cancellations are shown. The definition 1 cm = 10 mm may be used for *mm* to *cm* conversions. Doing so will change the number of conversions but not the answer.

1. ? cm = 12.0 inches • $2.54 \text{ cm} = 12.0 \cdot 2.54 = 30.5 \text{ cm}$ (check the cm on a 12 inch ruler) 1 inch

2. ? inches = 1.00 meters • 100 cm • 1 inch = 100 = 39.4 inches 1 meter 2.54 cm 2.54

3a. Aim to convert the *given* unit (mm) to the bridge unit that is in the same system (English or metric) as the *given*. **Cm** is the bridge unit in the same system as mm.

3b. ? inches = 760. mm • $1 \frac{1 \text{ meter}}{1000 \text{ mm}} \cdot \frac{100 \text{ cm}}{1 \text{ meter}} \cdot \frac{1 \text{ inch}}{2.54 \text{ cm}} = \frac{760}{10 \cdot 2.54} = 29.9 \text{ inches}$

<u>Sig figs</u>: 760., with the decimal after the 0, means 3 sig figs. Metric definitions and 1 have infinite sig figs. The answer must be rounded to 3 sig figs (see Module 2).

In countries that use English units for legal measure, 1 inch = 2.54 cm is now the *definition* of an inch. However, unless an equality is termed exact or a definition, you should assume that in conversions and equalities, a 1 is exact, but other numbers are precise only to the sig figs shown.

- 4. ? mm = 0.500 yd. <u>3 ft.</u> <u>12 in.</u> <u>2.54 cm</u> <u>1 meter</u> <u>1000 mm</u> = **457 mm** 1 yd. 1 ft. **1 inch** 100 cm 1 meter
- 5a. Aim to convert the *given* unit (miles) to the bridge unit in the same system (English or metric) as the *given*. **Inches** is in the same system as miles.
- 5b. ? km = 1 *mile* <u>5,280 ft.</u> <u>12 in.</u> <u>2.54 cm</u> <u>1 meter</u> <u>1 km</u> = 1.61 km 1 *mile* 1 ft. 1 inch 100 cm 1000 meter

Sig figs: Assume an integer 1 that is part of an equality or conversion is exact, with infinite sig figs.

6.
$$? \text{ grams} = 7.7 \text{ lbs} \cdot \underline{1 \text{ kg}} \cdot \underline{10^3 \text{ grams}} = 3.5 \times 10^3 \text{ grams}$$

2.2 lb 1 kg

<u>Sig figs</u>: 7.7 and 2.2 have 2 sig figs. A 1 has infinite sig figs. Definitions, including metric-prefix definitions, have infinite sig figs. Round the answer to 2 sig figs.

7. ? fluid ounces = 2.00 liters • <u>1000 mL</u> • <u>12.0 fl. oz.</u> = 67.6 fl. oz. <u>1 liter</u> <u>355 mL</u>

(Check this answer on any 2-liter soda bottle.)