

Forms and formulas for linear equations

General form of a line: $Ax + By = C$, where A , B , and C are *constants*, i.e. any real numbers.

Midpoint formula: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$, where (x_1, y_1) & (x_2, y_2) are any two points on the line.

Slope formula: $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) & (x_2, y_2) are any two points on the line.

Slope-intercept form: $y = mx + b$, m is the *slope* and $(0, b)$ is the *y-intercept*.

Point-slope form: $y - y_1 = m(x - x_1)$, m is the *slope* and (x_1, y_1) is a point on the line.

Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, where (x_1, y_1) & (x_2, y_2) are two points on the line.

Pythagorean formula: $a^2 + b^2 = c^2$, where a & b are the sides of a right triangle and c is the hypotenuse.

Some facts about linear equations

Positive slope: If $m > 0$, then the line falls to the left and rises to the right.

Negative slope: If $m < 0$, then the line falls to the right and rises to the left.

Vertical Lines: If a line is vertical, its *slope is undefined* and it is of the form $x = c$, where c is a constant.

Horizontal Lines: If a line is horizontal, its *slope is 0* and it is of the form $y = c$, where c is a constant.

Parallel lines: If two lines are parallel ($l_1 \parallel l_2$), then their slopes are equal, i.e. $m_1 = m_2$.

Perpendicular lines: If two lines are perpendicular ($l_1 \perp l_2$), then their slopes are the negative reciprocals of one another, i.e. $m_1 = \frac{-1}{m_2}$ or equivalently $m_1 \cdot m_2 = -1$.

Some facts about systems of linear equations (two equations)

Lines intersect at one point: Slopes are different, the *system is consistent* and the *lines* are independent since there is precisely **one solution**, which is a point on the coordinate plane.

Lines are parallel (do not intersect): Slopes are the same, *y-intercepts* are different, the *system is inconsistent* and the *lines* are independent since there is **no solution**, which we can denote as \emptyset .

Lines are the same: Slopes and *y-intercepts* are the same, the *system is consistent* and the *lines* are dependent since there are an **infinite number of solutions**, i.e. all real numbers, which we denote as $(-\infty, \infty)$.