The simplest atomic spectrum is that of the *hydrogen atom*. In 1886, Balmer showed that the lines in the spectrum of the hydrogen atom had wavelengths that could be expressed by a rather simple equation. In 1913, Bohr explained the spectrum on a theoretical basis with his famous model of the hydrogen atom. According to Bohr's theory, the energies E_n allowed to a hydrogen atom are all given by the following equation:

$$E_n = \frac{-B}{n^2} \tag{2}$$

where B is the constant, 2.178×10^{-18} J and n is an integer, 1, 2, 3,…, called a quantum number. It has been found that all the lines in the atomic spectrum of hydrogen can be associated with differences between atomic energy levels which are predicted with great accuracy by Bohr's equation.

There are several ways in which one might analyze an atomic spectrum, given the energy levels of the atom, but a simple and powerful method is to calculate the wavelengths of some of the lines that are allowed and see if they match those which are observed. We shall use this method in our workshop.

A. Energy Levels of Hydrogen

Given the expression for E_n in Equation 2, calculate the energy (in joules) for each of the levels of the H atom missing in the table below. Notice that the energies are all negative, so that the lowest energy will have the largest allowed negative value. Enter these values in the table of energy levels below:

Table One

Quantum Number	Energy, E_n, in joules	Quantum Number	Energy, E_n, in joules
1	-2.178×10^{-18} J	6	
2		7	
3		8	
4		10	-2.178×10^{-20} J
5		∞	ZERO Joules

(Workshop continued on next page)

B. Calculation of Wavelengths in the Spectrum of the Hydrogen Atom

The lines in the hydrogen spectrum all arise from jumps made by the atom from one energy level to another. The wavelengths in nanometers of these lines can be calculated by Equation 1, where $|\Delta E|$ is the positive difference in energy between any two allowed levels. By rearranging Equation 1 it is possible to solve for wavelengths:

$$\lambda = \frac{hc}{|\Delta E|} \tag{3}$$

After putting in some constants we can solve for wavelength in nanometers.

$$\lambda = \frac{(6.626 \times 10^{-34} \text{J·sec})(3.00 \times 10^{8} \text{ m / sec})}{|\Delta E|} \times \frac{1 \text{ nm}}{10^{-9} \text{ m}} \tag{4}$$

Calculate the $|\Delta E|$ and wavelength for all the jumps indicated in the table below. Write $|\Delta E|$, the difference in energy in J between $E_{n,hi}$ and $E_{n,lo}$, in the upper half of the box, and in the lower half of the box, write the λ (in nm) associated with that value. The box for the $n_2 \rightarrow n_1$ transition is filled in for you.

Table Two

n_{higher} / n_{lower}	6	5	4	3	2	
1					1.634×10^{-18}	$|\Delta E|$ (J)
1					121.6	λ (nm)
2						$|\Delta E|$ (J)
2						λ (nm)
3					$|\Delta E|$ (J)	
3					λ (nm)	
4				$|\Delta E|$ (J)		
4				λ (nm)		
5			$|\Delta E|$ (J)			
5			λ (nm)			

(Workshop continued on next page)

Name: _____   Section: _____

C. Assignment of Wavelengths

Compare the wavelengths you have calculated in Table Two with those listed in Table Three. You should notice that many wavelengths match within the error of your calculation. Fill in the quantum numbers of the upper and lower states for each line whose origin you can recognize by comparison of your calculated values with the observed values. Several wavelengths will not match at all; place those in Table Four and estimate the expected $n_{hi} \rightarrow n_{lo}$ following the trends. Note that Table Two only covers transitions with n_{hi} less than or equal to six. Check your estimations by solving for ΔE and wavelengths as you did on Table Two using n_{hi} numbers greater than six.

Table Three

Wavelength	$n_{hi} \rightarrow n_{lo}$	Wavelength	$n_{hi} \rightarrow n_{lo}$	Wavelength	$n_{hi} \rightarrow n_{lo}$
97.25		410.17		1,005.0	
102.57		434.05		1,093.8	
121.57	$2 \rightarrow 1$	486.13		1,281.8	
388.91		656.28		1,875.1	
397.01		954.62		4,050.0	

Table Four: Wavelengths you cannot assign using Table Two data

Observed Wavelength (in nm)	Predicted transition $n_{hi} \rightarrow n_{lo}$	Calculated ΔE (in J)	Calculated Wavelength λ (in nm)
388.91			

(Workshop continued on next page)

Name: _____   Section: _____

D. The Balmer Series

When Balmer formulated his famous series for hydrogen in 1886, he was limited experimentally to wavelengths for the visible and near ultraviolet regions from 250 nm to 700 nm. All the lines in his series lie in this wavelength range. All transitions in the Balmer Series have $n_{final} = 2$.

1. What would be the longest POSSIBLE wavelength for a line in the Balmer Series?

_____nm

2. What would be the shortest POSSIBLE wavelength for a line in the Balmer Series?

_____ nm

In a normal hydrogen atom, the electron is in the *lowest* energy state. The maximum energy of an electron in the hydrogen atom is 0 J, at which point the electron is in the $n = \infty$ state, essentially removed from the atom. At this point, ionization has occurred.

3. How much energy in joules does it take to ionize the hydrogen atom?

_____ J

4. The ionization energy you have calculated is for one electron in a single hydrogen atom. Calculate the ionization energy for one mole of H atoms.
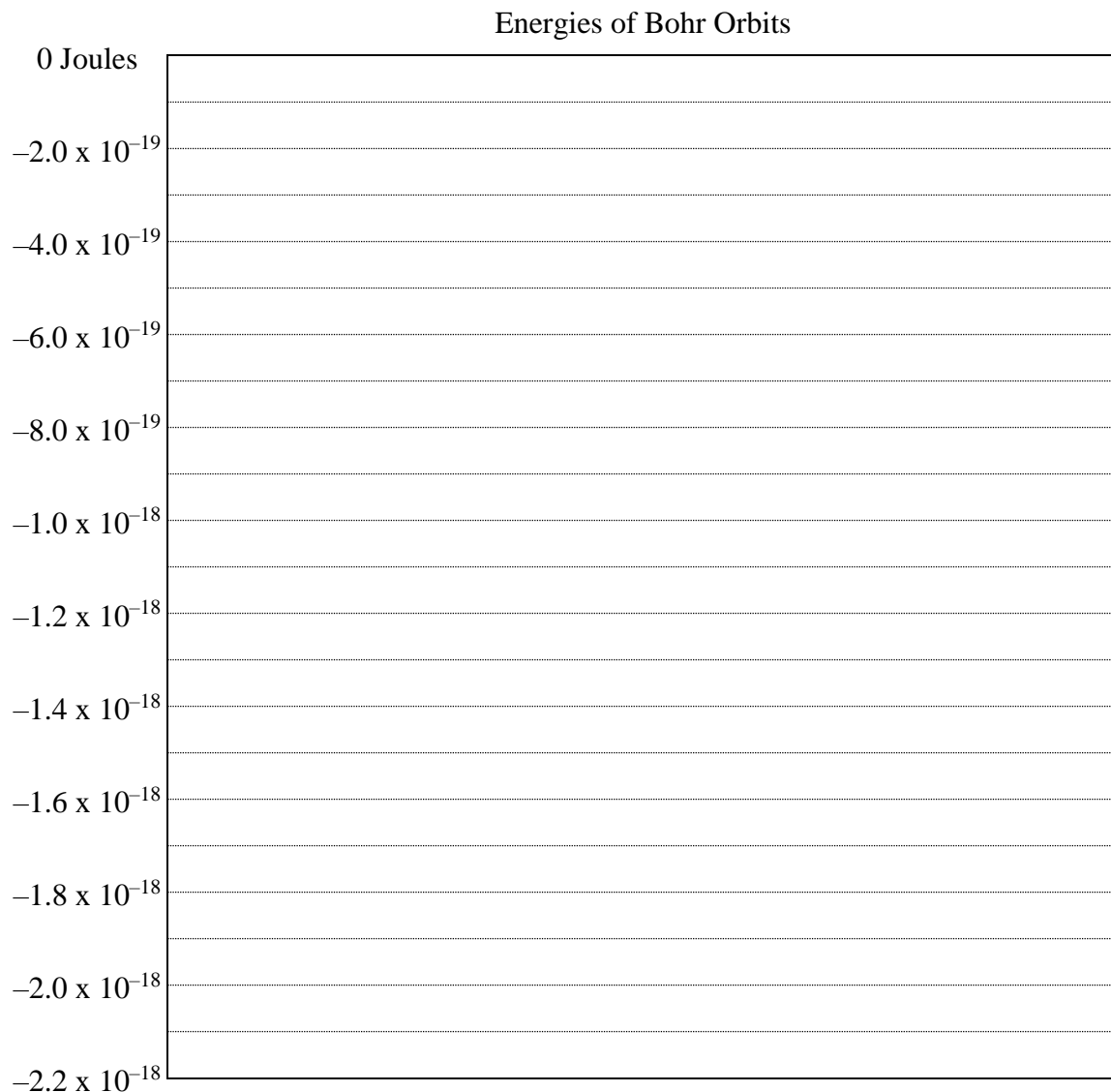
SHOW CALCULATIONS:

_____ kJ / mole

E. Energy Levels of Hydrogen Atom

Show each of the first six lowest energy states in the chart below using the values from Table One. Draw horizontal lines for each level and identify them by writing its quantum number on the right side. Use vertical arrows to show the electron transitions calculated in Table Three.

Energies of Bohr Orbits

0 Joules	
-2.0×10^{-19}	
-4.0×10^{-19}	
-6.0×10^{-19}	
-8.0×10^{-19}	
-1.0×10^{-18}	
-1.2×10^{-18}	
-1.4×10^{-18}	
-1.6×10^{-18}	
-1.8×10^{-18}	
-2.0×10^{-18}	
-2.2×10^{-18}	

F. Hydrogen Line Spectra

Draw the line spectra for hydrogen as it would appear in the visible region showing the lines calculated in Table Three within the Balmer Series.

700 nm   250 nm