

# MATH M21: DISCRETE MATHEMATICS

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**Originator**

pabramoff

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**College**

Moorpark College

**Discipline (CB01A)**

MATH - Mathematics

**Course Number (CB01B)**

M21

**Course Title (CB02)**

Discrete Mathematics

**Banner/Short Title**

Discrete Mathematics

**Credit Type**

Credit

**Start Term**

Fall 2023

**Catalog Course Description**

Covers elements of discrete mathematics which have application to computer science. Includes the following topics: logic, sets, functions, relations, proof techniques, mathematical induction, recurrence relations, graphs, trees, discrete probability, Boolean algebra and a brief introduction to programming.

Credit Limitations: MC, CSU and UC: MATH M21 and CS M155 combined: Maximum credit, one course.

**Taxonomy of Programs (TOP) Code (CB03)**

1701.00 - Mathematics, General

**Course Credit Status (CB04)**

D (Credit - Degree Applicable)

**Course Transfer Status (CB05) (select one only)**

A (Transferable to both UC and CSU)

**Course Basic Skills Status (CB08)**

N - The Course is Not a Basic Skills Course

**SAM Priority Code (CB09)**

E - Non-Occupational

**Course Cooperative Work Experience Education Status (CB10)**

N - Is Not Part of a Cooperative Work Experience Education Program

**Course Classification Status (CB11)**

Y - Credit Course

**Educational Assistance Class Instruction (Approved Special Class) (CB13)**

N - The Course is Not an Approved Special Class

**Course Prior to Transfer Level (CB21)**

Y - Not Applicable

**Course Noncredit Category (CB22)**

Y - Credit Course

**Funding Agency Category (CB23)**

Y - Not Applicable (Funding Not Used)

**Course Program Status (CB24)**

1 - Program Applicable

**General Education Status (CB25)**

B - Satisfies Math/Quantitative Reasoning req (CSUGE-B B4, IGETC 2, or 4-yr)

**Support Course Status (CB26)**

N - Course is not a support course

**Field trips**

Will not be required

**Grading method**

(L) Letter Graded

**Alternate grading methods**

(O) Student Option- Letter/Pass

(P) Pass/No Pass Grading

**Does this course require an instructional materials fee?**

No

**Repeatable for Credit**

No

**Is this course part of a family?**

No

**Units and Hours**

**Carnegie Unit Override**

No

**In-Class**

**Lecture**

**Minimum Contact/In-Class Lecture Hours**

52.5

**Maximum Contact/In-Class Lecture Hours**

52.5

**Activity**

**Laboratory**

**Total in-Class**

**Total in-Class**

**Total Minimum Contact/In-Class Hours**

52.5

**Total Maximum Contact/In-Class Hours**

52.5

**Outside-of-Class****Internship/Cooperative Work Experience**

Paid

Unpaid

**Total Outside-of-Class****Total Outside-of-Class****Minimum Outside-of-Class Hours**

105

**Maximum Outside-of-Class Hours**

105

**Total Student Learning****Total Student Learning****Total Minimum Student Learning Hours**

157.5

**Total Maximum Student Learning Hours**

157.5

**Minimum Units (CB07)**

3

**Maximum Units (CB06)**

3

**Prerequisites**

MATH M25A or MATH M25AH

**Advisories on Recommended Preparation**

CS M10A or CS M125

**Entrance Skills****Entrance Skills**

MATH M25A OR MATH M25AH

**Prerequisite Course Objectives**

MATH M25A-evaluate the limit of a function using numerical and algebraic techniques, the properties of limits, and analysis techniques.

MATH M25A-evaluate one-sided and two-sided limits for algebraic and trigonometric functions.

MATH M25A-determine analytically whether a limit fails to exist.

MATH M25A-determine whether a function is continuous or discontinuous at a point.

MATH M25A-apply the Intermediate Value Theorem to a continuous function on a closed interval.

MATH M25A-use the formal definition of the derivative to find the derivative of an algebraic function.

MATH M25A-apply the basic rules of differentiation to find the derivative of a function including the constant, power, sum, product, and quotient rules and The Chain Rule.

MATH M25A-find first-order and higher-order derivatives of algebraic and transcendental functions and their inverses.

MATH M25A-find the derivatives of functions and relations using implicit differentiation.

MATH M25A-solve applied problems using the derivative including rates of change, the tangent line problem, and related rates.

MATH M25A-apply the method of logarithmic differentiation for finding derivatives.

MATH M25A-demonstrate an understanding of the connection between differentiability and continuity of a function.

MATH M25A-apply Rolle's Theorem and the Mean Value Theorem to a function on a closed interval.

MATH M25A-identify indeterminate forms and use L'Hospital's Rule to evaluate limits.

MATH M25A-apply analytic techniques to a function and its derivatives to solve curve sketching problems.

MATH M25A-use differentials with linear approximation problems.

MATH M25A-solve applied optimization problems.  
 MATH M25A-find an approximate solution to an equation using Newton's Method (optional).  
 MATH M25A-apply the basic rules of integration for finding anti-derivatives for algebraic and transcendental functions.  
 MATH M25A-use summation notation with Riemann sums and upper and lower sums.  
 MATH M25A-use the formal definition of the definite integral to evaluate the integral of an algebraic function over a closed interval.  
 MATH M25A-evaluate definite integrals using the properties of integrals and the Fundamental Theorem of Calculus.  
 MATH M25A-integrate indefinite and definite integrals using change of variable techniques.  
 MATH M25A-use integration and analysis techniques to find the area of a region between two curves.  
 MATH M25A-solve exponential growth and decay problems (optional).

## Requisite Justification

### Requisite Type

Prerequisite

### Requisite

MATH M25A OR MATH M25AH

### Requisite Description

Course in a sequence

### Level of Scrutiny/Justification

Required by 4 year institution

### Requisite Type

Recommended Preparation

### Requisite

CS M125 (CS M10A)

### Requisite Description

Course not in a sequence

## Student Learning Outcomes (CSLOs)

**Upon satisfactory completion of the course, students will be able to:**

- 1 identify one-to-one (injective) and onto (surjective) functions.
- 2 use mathematical induction to prove a number theoretic proposition.
- 3 perform a traversal on a binary tree.

## Course Objectives

**Upon satisfactory completion of the course, students will be able to:**

- 1 simplify and evaluate basic logic statements including compound statements, implications, inverses, converses, and contrapositives using truth tables and the properties of logic.
- 2 express a logic sentence in terms of predicates, quantifiers, and logical connectives.
- 3 apply the operations of sets and use Venn diagrams to solve applied problems; solve problems using the principle of inclusion-exclusion.
- 4 determine the domain and range of a discrete or non-discrete function, graph functions, identify one-to-one (injective) functions, onto (surjective) functions, perform the composition of functions, find and/or graph the inverse of a function, and apply the properties of functions to application problems.
- 5 list the terms in a sequence, write a sequence in closed form, compute the sum of a finite sequence, compute the product of a finite sequence, and express sequences in terms of recursive or non-recursive forms; understand the relationship between sequences and strings.
- 6 analyze the growth of elementary functions and determine their Big-O, Big Omega, and Big-Theta values; analyze simple algorithms and compare two algorithms based on computational complexity.

- 7 use elementary number theory including the divisibility properties of numbers to determine prime numbers and composites, the greatest common divisor, and the least common multiple; perform modulo arithmetic and computer arithmetic.
- 8 apply algorithms to problems including searching algorithms, base conversion algorithms, and the Euclidean algorithm.
- 9 perform basic matrix operations including sums, products, and transpose and perform 0-1 matrix operations.
- 10 apply rules of inference, tests for validity, and methods of proof including direct and indirect proof forms, proof by contradiction, proof by cases, and mathematical induction and write proofs using symbolic logic and Boolean Algebra.
- 11 identify the base step and the recursive or inductive step in applied problems and give a recursive and a non-recursive definition for an iterative algorithm.
- 12 verify that a simple program segment with given initial and final assertions is correct using the rule of inference for verification of partial correctness and loop invariants.
- 13 solve counting problems by applying elementary counting techniques using the product and sum rules, permutations, combinations, the pigeon-hole principle, and binomial expansion.
- 14 solve discrete probability problems and use sets to solve problems in combinatorics and probability theory.
- 15 solve problems using recurrence relations and recursion to analyze algorithms and programs such as finding Fibonacci numbers, the Ackerman function and Tower of Hanoi problems.
- 16 solve problems using divide-and-conquer recurrence relations such as the fast multiplication algorithm and binary search.
- 17 describe binary relations between two sets; determine if a binary relation is reflexive, symmetric, transitive, an equivalence relation, a partial ordering, or a whole ordering; combine relations using set operations and composition.
- 18 describe N-ary relations between N sets and apply basic database operations such as projections to N-ary relations.
- 19 determine if a given graph is simple or a multigraph, directed or undirected, cyclic or acyclic, and determine the connectivity of a graph.
- 20 represent a graph using an adjacency list and an adjacency matrix and apply graph theory to application problems such as computer networks.
- 21 determine if a graph has an Euler or a Hamilton path or circuit.
- 22 determine if a graph is a binary tree, N-ary tree, or not a tree; use the properties of trees to classify trees, identify ancestors, descendants, parents, children, and siblings; determine the level of a node, the height of a tree or subtree and apply counting theorems to the edges and vertices of a tree.
- 23 perform tree traversals using preorder, inorder, and postorder traversals and apply these traversals to application problems; use binary search trees or decision trees to solve problems.
- 24 evaluate Boolean functions and simplify expression using the properties of Boolean algebra; apply Boolean algebra to circuits and gating networks.
- 25 use finite-state machines to model computer operations.

## Course Content

### Lecture/Course Content

#### 18% A. Logic, Sets, and Functions

1. Formal Logic including Statements and Symbolic Representation
2. Propositional Logic, Equivalences, and Tautologies
3. Predicates, Validity, and Quantifiers
4. Logic Programming
5. Sets and Set Operations
6. Functions
7. Sequences and Summations
8. Recurrence Relations

#### 12% B. Algorithms, the Integers, and Matrices

1. Algorithms and the Complexity of Algorithms
2. The Growth of Functions and Big-O Notation
3. The Integers, Division, and Modulo Arithmetic
4. Integers and Algorithms
5. Number Theory
6. Matrices

#### 15% C. Mathematical Reasoning

1. Methods of Proof

- 2. Mathematical Induction
- 3. Recursive Definitions and Algorithms
- 4. Program Correctness and Loop Invariants

**15% D. Combinatorics and Probability**

- 1. The Basics of Counting
- 2. The Pigeonhole Principle
- 3. Permutations and Combinations
- 4. Binomial Coefficients and the Binomial Theorem
- 5. Discrete Probability
- 6. Divide-and-Conquer Algorithms and Recurrence Relations
- 7. Inclusion-Exclusion

**7% E. Relations**

- 1. Relations and their Properties
- 2. N-ary Relations and their Applications
- 3. Representing Relations
- 4. Equivalence Relations
- 5. Closure Relations, Transitive Closure and Warshall's Algorithm

**10% F. Graphs**

- 1. Introduction to Graphs and Graph Terminology
- 2. Directed and Undirected Graphs
- 3. Representing Graphs and Graph Isomorphism
- 4. Connectivity and Articulation Points
- 5. Euler and Hamilton Paths and Circuits
- 6. Graph Applications including Computer Networks

**8% G. Trees**

- 1. Introduction to Trees
- 2. Applications of Trees including:
  - a. Binary and N-ary Trees
  - b. Decision Trees
  - c. Expression Trees
  - d. Huffman Codes
- 3. Tree Traversals
- 4. Trees and Searching and Sorting Algorithms
- 5. Minimal Spanning Trees

**8% H. Boolean Algebra**

- 1. Boolean Functions
- 2. Representing Boolean Functions
- 3. Logic Gates
- 4. Minimization of Circuits

**7% I. Modeling Computation**

- 1. Introduction to Languages and Grammars
- 2. The Algebraic Structure of Phrase-Structure Grammars
- 3. Finite-State Machines and Formal Languages

**Laboratory or Activity Content**

n/a

**Methods of Evaluation**

**Which of these methods will students use to demonstrate proficiency in the subject matter of this course? (Check all that apply):**

- Written expression
- Problem solving exercises
- Skills demonstrations

**Methods of Evaluation may include, but are not limited to, the following typical classroom assessment techniques/required assignments (check as many as are deemed appropriate):**

- Computational homework
- Individual projects
- Mathematical proofs
- Objective exams
- Problem-solving exams
- Problem-solving homework
- Quizzes
- Other (specify)
- Classroom Discussion

## Projects

### Other

Quizzes and graded work will be used to evaluate students for the critical thinking skills needed to solve math problems. Problems must require students to demonstrate analytic skills and the step-by-step details required for the solution.

## Instructional Methodology

### Specify the methods of instruction that may be employed in this course

Class activities  
 Class discussions  
 Collaborative group work  
 Computer-aided presentations  
 Demonstrations  
 Distance Education  
 Group discussions  
 Instructor-guided interpretation and analysis  
 Instructor-guided use of technology  
 Lecture  
 Problem-solving examples  
 Other (specify)

### Specify other method of instruction

All instructors will use best practices to provide an inclusive learning environment that respects all forms of racial, ethnic, age, and gender diversity, and provides for the individual needs of students of all learning styles.

### Describe specific examples of the methods the instructor will use:

Introductory lectures to new concepts; review material from previous topics as related to the current topic; provide detailed step-by-step examples; provide practice problems to develop proper mathematical skills and techniques; provide student interaction for questions and answers; use projects and/or group work to enhance student understanding of the concepts; and discuss application problems.

## Representative Course Assignments

### Writing Assignments

1. Homework problems selected from the discrete mathematics textbook where answers require a written explanation of the solution such as solving a problem using the Pigeonhole Principle.
2. Short answer problems on exams such as stating the results for an application problem.
3. Graded assignments: in-class and/or homework assignments requiring complete solutions using both written English and symbolic mathematical language.

### Critical Thinking Assignments

1. Describe and apply the algorithmic steps for obtaining the solution to a discrete mathematics problem such as solving a problem using mathematical induction.
2. Compare and contrast methods of solution to discrete mathematics problems such as comparing two algorithms based on their computational complexity.
3. Apply analytic techniques for solving mathematical and application problems such as using trees to solve Huffman code problems.

### Reading Assignments

1. Read and understand the properties of mathematical concepts that studied in the class.
2. Read scholarly and/or news articles on how discrete mathematics can be used for the social good.

### Skills Demonstrations

1. Use proof by contradiction to show that the square root of 2 is an irrational number
2. Interpret a discrete probability distribution.
3. Complete practice exercises on boolean algebras.

**Problem-Solving and Other Assignments (if applicable)**

1. Problem-solving exercises to practice basic matrix operations including sums, products, and transpose and perform 0-1 matrix operations.
2. Problem-solving exercises using recurrence relations and recursion to analyze algorithms and programs such as finding Fibonacci numbers, the Ackerman function and Tower of Hanoi problems.

**Outside Assignments****Representative Outside Assignments**

1. Computer programming projects.
2. Group or individual discrete mathematics projects.
3. Graded problem solving assignments.
4. Additional problem sets provided by the instructor such as problems on logic, on discrete probability, or on the shortest path problem.
5. Assigned reading material and homework problems from the discrete mathematics textbook.

**Articulation****C-ID Descriptor Number**

MATH M160

**Status**

Approved

**Equivalent Courses at 4 year institutions**

University	Course ID	Course Title	Units
UC Berkeley	MATH 55	Discrete Mathematics	4
UC Los Angeles	MATH 61	Introduction to Discrete Structures	4
Cal Poly, Humboldt	MATH	Discrete Structures I	3
CSU Dominguez Hills	MAT 281	Discrete Mathematics	3

**Comparable Courses within the VCCCD**

CS V17 - Discrete Structures

MATH V52 - Discrete Structures

**Equivalent Courses at other CCCs**

College	Course ID	Course Title	Units
City College of San Francisco	MATH 115	Discrete Mathematics	3
San Jose City College	MATH 070	Discrete Mathematics	4
Las Positas College	MATH 10	Discrete Mathematics Structures	4

**District General Education****A. Natural Sciences****B. Social and Behavioral Sciences****C. Humanities****D. Language and Rationality****D2. Communication/Analytical Thinking**

Approved



**E. Health and Physical Education/Kinesiology****F. Ethnic Studies/Gender Studies****Course is CSU transferable**

Yes

**CSU Baccalaureate List effective term:**

F2000

**CSU GE-Breadth****Area A: English Language Communication and Critical Thinking****Area B: Scientific Inquiry and Quantitative Reasoning****B4 Mathematical/Quantitative Reasoning**

Approved

**Area C: Arts and Humanities****Area D: Social Sciences****Area E: Lifelong Learning and Self-Development****Area F: Ethnic Studies****CSU Graduation Requirement in U.S. History, Constitution and American Ideals:****UC TCA****UC TCA**

Approved

**IGETC****Area 1: English Communication****Area 2A: Mathematical Concepts & Quantitative Reasoning****Area 2A: Mathematical Concepts & Quantitative Reasoning**

Approved

**Area 3: Arts and Humanities****Area 4: Social and Behavioral Sciences****Area 5: Physical and Biological Sciences****Area 6: Languages Other than English (LOTE)****Textbooks and Lab Manuals****Resource Type**

Textbook

**Classic Textbook**

No

**Description**

Rosen, Kenneth. *Discrete Mathematics and Its Applications*. 8th, McGraw-Hill, 2019.

**Resource Type**

Textbook

**Classic Textbook**

No

**Description**

Johnsonbaugh, Richard. *Discrete Mathematics*. 8th ed., Pearson, 2023.

**Resource Type**

Textbook

**Classic Textbook**

Yes

**Description**

Goodaire, Edgar, and Michael Parmenter. *Discrete Mathematics with Graph Theory*. 3rd ed., Pearson, 2023.

**Resource Type**

Textbook

**Description**

Levin, Oscar. *Discrete Mathematics: An Open Introduction*. E-book, 3rd ed., Open Textbook Library, 2016, <https://open.umn.edu/opentextbooks/textbooks/394>. Accessed on 20 Oct 2022.

**Library Resources****Assignments requiring library resources**

Research using the Library's print and online resources. If the student does not have their own computer, then possible use of the Library's for programming.

**Sufficient Library Resources exist**

Yes

**Example of Assignments Requiring Library Resources**

Using Library's print, online, and computer resources to locate readings on how discrete mathematics is applied by businesses and governments to solve problems.

**Distance Education Addendum****Definitions****Distance Education Modalities**

Hybrid (1%–50% online)  
Hybrid (51%–99% online)  
100% online

**Faculty Certifications**

**Faculty assigned to teach Hybrid or Fully Online sections of this course will receive training in how to satisfy the Federal and state regulations governing regular effective/substantive contact for distance education. The training will include common elements in the district-supported learning management system (LMS), online teaching methods, regular effective/substantive contact, and best practices.**

Yes

Faculty assigned to teach Hybrid or Fully Online sections of this course will meet with the EAC Alternate Media Specialist to ensure that the course content meets the required Federal and state accessibility standards for access by students with disabilities. Common areas for discussion include accessibility of PDF files, images, captioning of videos, Power Point presentations, math and scientific notation, and ensuring the use of style mark-up in Word documents.

Yes

## Regular Effective/Substantive Contact

### Hybrid (1%–50% online) Modality:

Method of Instruction	Document typical activities or assignments for each method of instruction
Asynchronous Dialog (e.g., discussion board)	Use of student discussion boards to discuss concepts from the material, solutions to homework problems, general discussion of techniques in solving problems, study skills, or arranging study groups.
E-mail	Responding to student queries about material, grade information, course policies and procedures, scheduling and due dates, submitting homework assignments, or making general announcements to the class.
Face to Face (by student request; cannot be required)	Students requesting to speak to instructor in person for personal help on material, grade information, or discussion of policies and procedures.
Other DE (e.g., recorded lectures)	Posting of recorded lectures either by the instructor, recorded lessons available through campus resources, or use of public online resources available on the internet.
Synchronous Dialog (e.g., online chat)	Active live discussion with the instructor on material concepts, techniques for problem solving, feedback on solutions to problems, general chat on study skills, or answers to homework problems, quizzes or tests.

### Hybrid (51%–99% online) Modality:

Method of Instruction	Document typical activities or assignments for each method of instruction
Asynchronous Dialog (e.g., discussion board)	Use of student discussion boards to discuss concepts from the material, solutions to homework problems, general discussion of techniques in solving problems, study skills, or arranging study groups.
E-mail	Responding to student queries about material, grade information, course policies and procedures, scheduling and due dates, submitting homework assignments, or making general announcements to the class.
Face to Face (by student request; cannot be required)	Students requesting to speak to instructor in person for personal help on material, grade information, or discussion of policies and procedures.
Other DE (e.g., recorded lectures)	Posting of recorded lectures either by the instructor, recorded lessons available through campus resources, or use of public online resources available on the internet.
Synchronous Dialog (e.g., online chat)	Active live discussion with the instructor on material concepts, techniques for problem solving, feedback on solutions to problems, general chat on study skills, or answers to homework problems, quizzes or tests.

### 100% online Modality:

Method of Instruction	Document typical activities or assignments for each method of instruction
Asynchronous Dialog (e.g., discussion board)	Use of student discussion boards to discuss concepts from the material, solutions to homework problems, general discussion of techniques in solving problems, study skills, or arranging study groups.
E-mail	Responding to student queries about material, grade information, course policies and procedures, scheduling and due dates, submitting homework assignments, or making general announcements to the class.
Other DE (e.g., recorded lectures)	Posting of recorded lectures either by the instructor, recorded lessons available through campus resources, or use of public online resources available on the internet.

Synchronous Dialog (e.g., online chat)

Active live discussion with the instructor on material concepts, techniques for problem solving, feedback on solutions to problems, general chat on study skills, or answers to homework problems, quizzes or tests.

## Examinations

### Hybrid (1%–50% online) Modality

On campus

### Hybrid (51%–99% online) Modality

On campus

## Primary Minimum Qualification

MATHEMATICS

## Review and Approval Dates

### Department Chair

10/14/2022

### Dean

10/17/2022

### Technical Review

10/20/2022

### Curriculum Committee

11/01/2022

### DTRW-I

MM/DD/YYYY

### Curriculum Committee

MM/DD/YYYY

### Board

MM/DD/YYYY

### CCCCO

MM/DD/YYYY

### Control Number

CCC000434757

### DOE/accreditation approval date

MM/DD/YYYY