MATH M31: INTRODUCTION TO LINEAR ALGEBRA

Originator
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College
Moorpark College

Discipline (CB01A)
MATH - Mathematics

Course Number (CB01B)
M31

Course Title (CB02)
Introduction To Linear Algebra

Banner/Short Title
Intro To Linear Algebra

Credit Type
Credit

Start Term
Fall 2023

Catalog Course Description
Develops the techniques and theory needed to solve and classify systems of linear equations. Covers solution techniques including row operations, Gaussian elimination, and matrix algebra. Investigates the properties of vectors in two and three dimensions, leading to the notion of an abstract vector space. Presents vector space and matrix theory including topics such as inner products, norms, orthogonality, eigenvalues, eigenspaces, and linear transformations. Involves selected applications of linear algebra.

Taxonomy of Programs (TOP) Code (CB03)
1701.00 - Mathematics, General

Course Credit Status (CB04)
D (Credit - Degree Applicable)

Course Transfer Status (CB05) (select one only)
A (Transferable to both UC and CSU)

Course Basic Skills Status (CB08)
N - The Course is Not a Basic Skills Course

SAM Priority Code (CB09)
E - Non-Occupational

Course Cooperative Work Experience Education Status (CB10)
N - Is Not Part of a Cooperative Work Experience Education Program

Course Classification Status (CB11)
Y - Credit Course

Educational Assistance Class Instruction (Approved Special Class) (CB13)
N - The Course is Not an Approved Special Class

Course Prior to Transfer Level (CB21)
Y - Not Applicable

Course Noncredit Category (CB22)
Y - Credit Course

Funding Agency Category (CB23)
Y - Not Applicable (Funding Not Used)

Course Program Status (CB24)
1 - Program Applicable

General Education Status (CB25)
B - Satisfies Math/Quantitative Reasoning req (CSUGE-B B4, IGETC 2, or 4-yr)

Support Course Status (CB26)
N - Course is not a support course

Field trips
Will not be required

Grading method
(L) Letter Graded

Alternate grading methods
(O) Student Option- Letter/Pass
(P) Pass/No Pass Grading

Does this course require an instructional materials fee?
No

Repeatable for Credit
No

Is this course part of a family?
No

Units and Hours

Carnegie Unit Override
No

In-Class

Lecture
Minimum Contact/In-Class Lecture Hours
52.5
Maximum Contact/In-Class Lecture Hours
52.5

Activity

Laboratory

Total in-Class

Total in-Class
Total Minimum Contact/In-Class Hours
52.5
Total Maximum Contact/In-Class Hours
52.5

Outside-of-Class

Internship/Cooperative Work Experience

Paid

Unpaid

Total Outside-of-Class

Total Outside-of-Class
Minimum Outside-of-Class Hours
105
Maximum Outside-of-Class Hours
105

Total Student Learning

Total Student Learning
Total Minimum Student Learning Hours
157.5
Total Maximum Student Learning Hours
157.5

Minimum Units (CB07)
3
Maximum Units (CB06)
3

Prerequisites
MATH M25B or MATH M25BH or equivalent course

Entrance Skills

Entrance Skills
MATH M25B OR MATH M25BH

Prerequisite Course Objectives
MATH M25B-apply the definite integral to solve problems involving area, volume, arc length, surface area, work, moments and centers of mass in the plane, fluid force, and other applications.
MATH M25B-select appropriate techniques for evaluating an indefinite integral; techniques include basic integration forms, change of variables, integration by parts, trigonometric identities, trigonometric substitutions, partial fractions, and estimation techniques.
MATH M25B-identify and evaluate improper integrals using correct limit notation.
MATH M25B-determine the convergence or divergence of an infinite sequence using analytic techniques.
MATH M25B-determine whether a sequence is bounded or is monotonic.
MATH M25B-compute partial sums for infinite series.
MATH M25B-recognize telescoping, geometric, and p-series.
MATH M25B-determine the convergence or divergence of a geometric series and p-series.
MATH M25B-compute the sum of a convergent telescoping series and geometric series.

MATH M25B-apply appropriate tests which include the nth-term test, integral test, comparison tests, and ratio and root tests to determine the convergence or divergence of positive term series.
MATH M25B-apply the algebraic properties of infinite series.
MATH M25B-apply the alternating series test and analyze the remainder of an alternating series.
MATH M25B-determine if a series converges absolutely, conditionally, or if the series diverges.
MATH M25B-determine the interval of convergence for a power series using analysis techniques and the tests for convergence.
MATH M25B-apply integration and differentiation techniques for finding power series of elementary functions.
MATH M25B-compute Taylor and Maclaurin polynomial approximations of elementary functions with remainder.
MATH M25B-compute the power series for an elementary function centered at a point.
MATH M25B-sketch the graph of a parametric curve and indicate its orientation.
MATH M25B-convert the equation of a curve given in parametric form to rectangular form and vice versa.
MATH M25B-find the slope of the tangent line at a point on a curve given in parametric form.
MATH M25B-compute the arc length of a curve and the area of a surface of revolution for curves given in parametric form.
MATH M25B-sketch the graph of a polar equation.
MATH M25B-convert the equation of a curve given in polar form to rectangular form and vice versa.
MATH M25B-find the slope of the tangent line at a point on a curve given in polar form.
MATH M25B-find the area of a region bounded by a polar equation.
MATH M25B-find the arc length of a curve given in polar form.
MATH M25B-compute and identify the critical information for the standard conics, such as the vertex, directrix, and axis of symmetry for a parabola; the foci and vertices of an ellipse; and the vertices, foci, and asymptotes of a hyperbola.
MATH M25BH-apply the definite integral to solve problems involving area, volume, arc length, surface area, work, moments and centers of mass in the plane, fluid force.
MATH M25BH-select appropriate techniques for evaluating an indefinite integral; techniques include basic integration forms, change of variables, integration by parts, trigonometric identities, trigonometric substitutions, partial fractions, and estimation techniques.
MATH M25BH-identify and evaluate improper integrals using correct limit notation.
MATH M25BH-determine the convergence or divergence of an infinite sequence using analytic techniques.
MATH M25BH-determine whether a sequence is bounded or is monotonic.
MATH M25BH-compute partial sums for infinite series.
MATH M25BH-recognize telescoping, geometric, and p-series.
MATH M25BH-determine the convergence or divergence of a geometric series and p-series.
MATH M25BH-compute the sum of a convergent telescoping series and geometric series.
MATH M25BH-apply appropriate tests which include the nth-term test, integral test, comparison tests, and ratio and root tests to determine the convergence or divergence of positive term series.
MATH M25BH-apply the algebraic properties of infinite series.
MATH M25BH-apply the alternating series test and analyze the remainder of an alternating series.
MATH M25BH-determine if a series converges absolutely, conditionally, or if the series diverges.
MATH M25BH-determine the interval of convergence for a power series using analysis techniques and the tests for convergence.
MATH M25BH-apply integration and differentiation techniques for finding power series of elementary functions.
MATH M25BH-compute Taylor and Maclaurin polynomial approximations of elementary functions with remainder.
MATH M25BH-compute the power series for an elementary function centered at a point.
MATH M25BH-sketch the graph of a parametric curve and indicate its orientation.
MATH M25BH-convert the equation of a curve given in parametric form to rectangular form and vice versa.
MATH M25BH-find the slope of the tangent line at a point on a curve given in parametric form.
MATH M25BH-compute the arc length of a curve and the area of a surface of revolution for curves given in parametric form.
MATH M25BH-sketch the graph of a polar equation.
MATH M25BH-convert the equation of a curve given in polar form to rectangular form and vice versa.
MATH M25BH-find the slope of the tangent line at a point on a curve given in polar form.
MATH M25BH-find the area of a region bounded by a polar equation.
MATH M25BH-find the arc length of a curve given in polar form.
MATH M25BH-compute and identify the critical information for the standard conics, such as the vertex, directrix, and axis of symmetry for a parabola; the foci and vertices of an ellipse; and the vertices, foci, and asymptotes of a hyperbola.

Requisite Justification

Requisite Type

Prerequisite

Requisite

MATH M25B OR MATH M25BH

Requisite Description

Course in a sequence

Level of Scrutiny/Justification

Required by 4 year institution

Student Learning Outcomes (CSLOs)

	Upon satisfactory completion of the course, students will be able to:
1	find the matrix representation of a linear transformation given bases of the relevant vector spaces.
2	find the null space of a matrix and represent it as the span of independent vectors.
3	compute the inverse of an invertible matrix.

Course Objectives

	Upon satisfactory completion of the course, students will be able to:
1	solve systems of linear equations using various methods including Gaussian and Gauss-Jordan elimination and inverse matrices.
2	perform matrix algebra, invertibility, and the transpose and understand vector algebra in R n .
3	determine relationship between coefficient matrix invertibility and solutions to a system of linear equations and the inverse matrices.
4	define special matrices: diagonal, triangular, and symmetric.
5	comprehend determinants and their properties.
6	comprehend real vector spaces and subspaces and apply their properties.
7	comprehend linear independence and dependence.
8	find basis and dimension of a vector space, and understand change of basis.
9	find a basis for the row space, column space and null space of a matrix and find the rank and nullity of a matrix.
10	compute linear transformations, kernel and range, and inverse linear transformations, and find matrices of general linear transformations.
11	find the dimension of spaces such as those associated with matrices and linear transformations.
12	find eigenvalues and eigenvectors and use them in applications.
13	diagonalize and orthogonally diagonalize symmetric matrices.
14	evaluate the dot product, norm, angle between vectors, and orthogonality of two vectors in R n .
15	compute inner products on a real vector space and compute angle and orthogonality in inner product spaces.
16	create orthogonal and orthonormal bases: Gram-Schmidt process and use bases and orthonormal bases to solve application problems.
17	prove basic results in linear algebra using appropriate proof-writing techniques such as linear independence of vectors; properties of subspaces; linearity, injectivity and surjectivity of functions; and properties of eigenvectors and eigenvalues.

Course Content

Lecture/Course Content

10% A. Systems of Linear Equations
1. Representation and solution of systems of equations by Gaussian elimination and inverse matrices.

20% B. Matrices as Objects
1. Algebraic operations on matrices
2. Inverse matrices
3. Elementary matrices

5% C. Determinants
1. Methods for computing determinants
2. Applications of determinants

20% D. Vector Spaces
1. Definition and basic properties of vector spaces and subspaces
2. Span and linear independence/dependence of vectors
3. Bases of vector spaces and dimension
4. Row and column spaces
5. Rank and nullity of a matrix

15% E. Inner Product Spaces
1. Definition and basic properties of an inner product

2. Dot product, norm of a vector, and orthogonality
3. Gram-Schmidt process

15% F. Linear Transformations
1. Definition and basic properties of linear transformations
2. Kernel and range
3. Matrix representations and change of basis
4. Isomorphisms including one-to-one and onto linear transformations.

10% G. Eigenvalues and Eigenvectors
1. Basic definitions
2. Finding eigenvectors and eigenspaces

5% H. Proofs
1. Introduction to proofs and proof techniques
2. Direct, indirect, contrapositive, and proof by contradiction.

Laboratory or Activity Content

n/a

Methods of Evaluation

Which of these methods will students use to demonstrate proficiency in the subject matter of this course? (Check all that apply):

Written expression
Problem solving exercises
Skills demonstrations

Methods of Evaluation may include, but are not limited to, the following typical classroom assessment techniques/required assignments (check as many as are deemed appropriate):

Computational homework
Individual projects
Mathematical proofs
Objective exams
Problem-solving exams
Problem-solving homework
Quizzes
Other (specify)
Classroom Discussion
Projects
Participation

Other

Quizzes and/or graded work will be used to evaluate students for the critical thinking skills needed to solve math problems. Problems must require students to demonstrate analytic skills and the step-by-step details required for the solution.
Problems will include computational and proof-based questions, demanding formal algebraic techniques, and a geometric intuition for vector spaces and linear transformations.

Instructional Methodology

Specify the methods of instruction that may be employed in this course

Class discussions
Distance Education
Lecture
Problem-solving examples
Other (specify)

Specify other method of instruction

All instructors will use best practices to provide an inclusive learning environment that respects all forms of racial, ethnic, age, and gender diversity, and provides for the individual needs of students of all learning styles.

Describe specific examples of the methods the instructor will use:

Use of whiteboard or document camera to present lecture on theory of concepts and demonstration of computational examples.

Classroom discussion and student participation in discussing theory and methods of writing proofs.

Student individual and group work in attempting computational problems and writing rudimentary proofs.

Review material from previous topics as related to the current topic; provide detailed step-by-step examples; provide practice problems to develop proper mathematical skills and techniques; provide student interaction for questions and answers; use projects and/or group work to enhance student understanding of the concepts; and discuss application problems.

Representative Course Assignments

Writing Assignments

1. Homework problems selected from the linear algebra textbook where answers require a written explanation of the solution, such as solving a system of linear equations and identifying if the system is consistent or inconsistent, dependent or independent.
2. Short answer problems on exams, such as defining the basis of a vector space.
3. Graded assignments: in-class and/or homework assignments requiring complete solutions using both written English and symbolic mathematical language, such as a a written induction proof about the invertibility of a product of invertible matrices.

Critical Thinking Assignments

1. Describe and apply the algorithmic steps for obtaining the solution to a mathematical problem, such as using the Gram-Schmidt process to obtain an orthonormal basis.
2. Compare and contrast methods of solution to mathematical problems, such as determining the linear dependence or independence of a set of vectors by more than one method.
3. Apply analytic techniques for solving mathematical and application problems, such as finding the kernel, range and inverse of a linear transformation.

Reading Assignments

1. Read the textbook about Markov Chains to apply first order difference equations.
2. Read a paper resourced in the library, about QR factorizations.
3. Read a paper resourced in the library, about incidence matrices related to graphs.

Skills Demonstrations

1. Demonstrate the solution to a linear system by use of Gaussian elimination.
2. Demonstrate any of the various methods of determining the inverse of a square matrix.
3. Determine the matrix representation of a linear transformation relative to given bases of a vector space.

Problem-Solving and Other Assignments (if applicable)

1. Solve a linear system by use of Gaussian elimination.
2. Solve assigned graded assignments, such as projecting trends from algebraic models that involve homogeneous and nonhomogeneous linear systems.

Outside Assignments

Representative Outside Assignments

1. Group or individual projects, such as investigating the trajectories of vectors through iterated applications of a linear transformation, using a coordinate system relative to a basis of eigenvectors.
2. Additional problem sets provided by the instructor, such as practice exercises on finding the inverse of a matrix.
3. Assigned reading material from the linear algebra textbook, such as reading and mastering the Laplace Expansion Theorem for computing determinants.

Articulation

C-ID Descriptor Number

MATH 250

Status

Approved

Additional C-ID Descriptor(s)

C-ID Descriptor(s)	Status
MATH 910S with MATH M35	Approved

Equivalent Courses at 4 year institutions

University	Course ID	Course Title	Units
CSU Northridge	MATH 262	Introduction to Linear Algebra	3
CSU Long Beach	MATH 247	Introduction to Linear Algebra	3
CSU Los Angeles	MATH 2550	Linear Algebra	3
CSU Chico	MATH 235	Elementray Linear Algebra	3
Cal Poly Pomona	MAT 208	Introduction to Linear Algebra	4
UC Irvine	MATH 3A	Introduction to Linear Algebra	4
CSU Channel Islands	MATH 240	Linear Algebra	3
UCLA	MATH 33A	Linear Algebra and Applications	4

Comparable Courses within the VCCCD

MATH R134 - Linear Algebra
MATH V22 - Introduction to Linear Algebra

Equivalent Courses at other CCCs

College	Course ID	Course Title	Units
Santa Barbara City College	MATH 210	Linear Algebra	4
City College of San Francisco	MATH 120	Linear Algebra	3
Evergreen Valley College	MATH 079	Linear Algebra	3

District General Education

A. Natural Sciences

B. Social and Behavioral Sciences

C. Humanities

D. Language and Rationality

D2. Communication/Analytical Thinking

Approved

E. Health and Physical Education/Kinesiology

F. Ethnic Studies/Gender Studies

Course is CSU transferable

Yes

CSU GE-Breadth

Area A: English Language Communication and Critical Thinking

Area B: Scientific Inquiry and Quantitative Reasoning

B4 Mathematical/Quantitative Reasoning

Approved

Area C: Arts and Humanities

Area D: Social Sciences

Area E: Lifelong Learning and Self-Development

Area F: Ethnic Studies

CSU Graduation Requirement in U.S. History, Constitution and American Ideals:

UC TCA

UC TCA
Approved

IGETC

Area 1: English Communication

Area 2A: Mathematical Concepts & Quantitative Reasoning

Area 2A: Mathematical Concepts & Quantitative Reasoning
Approved

Area 3: Arts and Humanities

Area 4: Social and Behavioral Sciences

Area 5: Physical and Biological Sciences

Area 6: Languages Other than English (LOTE)

Textbooks and Lab Manuals

Resource Type
Textbook

Classic Textbook
No

Description
Anton, Howard, Chris Rorres, and Anton Kaul. *Elementary Linear Algebra: Applications Version.* 12th ed., Wiley, 2019.

Resource Type
Textbook

Classic Textbook
No

Description
Leon, Steve, and Lisette de Pillis. *Linear Algebra with Applications.* 10th ed., Pearson, 2020.

Resource Type
Textbook

Classic Textbook
No

Description
O'Leary, Michael L., *Linear Algebra.* Wiley, 2021.

Library Resources

Assignments requiring library resources
Use textbooks on reserve at Circulation Desk. Find a linear algebra application/article in a journal, to present in a class discussion, such as predator-prey models in difference equations, network flow in electrical circuits, or heat diffusion through a uniform surface.

Sufficient Library Resources exist
Yes

Example of Assignments Requiring Library Resources
Reading scientific books to identify examples of matrices and vector spaces that occur in real-life examples in physics, engineering, architecture, or earth science.

Distance Education Addendum

Definitions

Distance Education Modalities
Hybrid (1%–50% online)
Hybrid (51%–99% online)
100% online

Faculty Certifications

Faculty assigned to teach Hybrid or Fully Online sections of this course will receive training in how to satisfy the Federal and state regulations governing regular effective/substantive contact for distance education. The training will include common elements in the district-supported learning management system (LMS), online teaching methods, regular effective/substantive contact, and best practices.
Yes

Faculty assigned to teach Hybrid or Fully Online sections of this course will meet with the EAC Alternate Media Specialist to ensure that the course content meets the required Federal and state accessibility standards for access by students with disabilities. Common areas for discussion include accessibility of PDF files, images, captioning of videos, Power Point presentations, math and scientific notation, and ensuring the use of style mark-up in Word documents.
Yes

Regular Effective/Substantive Contact

Hybrid (1%–50% online) Modality:

Method of Instruction	Document typical activities or assignments for each method of instruction
Asynchronous Dialog (e.g., discussion board)	Use of student discussion boards to discuss concepts from the material, solutions to homework problems, general discussion of techniques in solving problems, study skills, or arranging study groups.
E-mail	Responding to student queries about material, grade information, course policies and procedures, scheduling and due dates, submitting homework assignments, or making general announcements to the class.
Face to Face (by student request; cannot be required)	Students requesting to speak to instructor in person for personal help on material, grade information, or discussion of policies and procedures.
Other DE (e.g., recorded lectures)	Posting of recorded lectures either by the instructor, recorded lessons available through campus resources, or use of public online resources available on the internet.
Synchronous Dialog (e.g., online chat)	Active live discussion with the instructor on material concepts, techniques for problem solving, feedback on solutions to problems, general chat on study skills, or answers to homework problems, quizzes or tests.

Hybrid (51%–99% online) Modality:

Method of Instruction	Document typical activities or assignments for each method of instruction
Asynchronous Dialog (e.g., discussion board)	Use of student discussion boards to discuss concepts from the material, solutions to homework problems, general discussion of techniques in solving problems, study skills, or arranging study groups.
E-mail	Responding to student queries about material, grade information, course policies and procedures, scheduling and due dates, submitting homework assignments, or making general announcements to the class.
Face to Face (by student request; cannot be required)	Students requesting to speak to instructor in person for personal help on material, grade information, or discussion of policies and procedures.
Other DE (e.g., recorded lectures)	Posting of recorded lectures either by the instructor, recorded lessons available through campus resources, or use of public online resources available on the internet.
Synchronous Dialog (e.g., online chat)	Active live discussion with the instructor on material concepts, techniques for problem solving, feedback on solutions to problems, general chat on study skills, or answers to homework problems, quizzes or tests.

100% online Modality:

Method of Instruction	Document typical activities or assignments for each method of instruction
Asynchronous Dialog (e.g., discussion board)	Use of student discussion boards to discuss concepts from the material, solutions to homework problems, general discussion of techniques in solving problems, study skills, or arranging study groups.
E-mail	Responding to student queries about material, grade information, course policies and procedures, scheduling and due dates, submitting homework assignments, or making general announcements to the class.
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Synchronous Dialog (e.g., online chat)	Active live discussion with the instructor on material concepts, techniques for problem solving, feedback on solutions to problems, general chat on study skills, or answers to homework problems, quizzes or tests.

Examinations

Hybrid (1%–50% online) Modality

On campus

Hybrid (51%–99% online) Modality

On campus

Primary Minimum Qualification

MATHEMATICS

Review and Approval Dates

Department Chair

MM/DD/YYYY

Dean

MM/DD/YYYY

Technical Review

10/06/2022

Curriculum Committee
10/18/2022

DTRW-I
MM/DD/YYYY

Curriculum Committee
MM/DD/YYYY

Board
MM/DD/YYYY

CCCCO
MM/DD/YYYY

Control Number
CCC000426892

DOE/accreditation approval date
MM/DD/YYYY