Module 1 – Scientific Notation

Timing

Module 1 should be done as soon as you are assigned problems that use exponential notation. If possible, do these lessons before textbook problems.

Each lesson has a pretest. If you pass the pretest, you may skip the lesson.

Additional Math Topics

Calculations involving *powers* and *roots* of scientific and exponential notation are covered in Lesson 25B.

Simplification of complex units such as → is covered in Lesson 17C.

$$\frac{\text{atm} \bullet \text{L}}{(\text{mole})\left(\frac{\text{atm} \bullet \text{L}}{\text{mole} \bullet \text{K}}\right)}$$

Lessons 25B and 17C may be done at any time after Module 1.

Calculators and Exponential Notation

To multiply 4.92 x 7.36, the calculator is a useful tool. However, when using exponential notation, you will make fewer mistakes if you do as much exponential math as you can with*out* a calculator. These lessons will review the rules for doing exponential math "in your head."

The majority of problems in Module 1 will *not* require a calculator. Problems that require a calculator will be clearly identified.

You are encouraged to try the more complex problems with the calculator *after* you have tried them without. This should help in deciding when, and when not, to use a calculator.

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Lesson 1A: Moving the Decimal

Pretest: If you get a perfect score on this pretest, you may skip to Lesson 1B. Otherwise, complete Lesson 1A. In these lessons, unless otherwise noted, answers are at the end of each lesson.

Change these to *scientific* notation.

a. $9{,}400 \times 10^{3}$ = ____________________

c. 0.042×10^{6} = __________________

b. 0.0067×10^{-2} = __________________

d. 77 = __________________

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Powers of 10

The chart below shows numbers that correspond to powers of 10. Note the change in the exponents and the numbers as you go down the sequence.

10^{6} = 1,000,000
10^{3} = 1,000 = 10 x 10 x 10
10^{2} = 100
10^{1} = 10
10^{0} = **1** (Anything to the zero power equals one.)
10^{-1} = 0.1
10^{-2} = 0.01 = $1/10^{2}$ = 1/100
10^{-3} = 0.001

Note also that the *number* in a positive power of 10 is equal to the number of

- *zeros* after the 1 in the corresponding number, and
- *places* that the decimal has moved to the *right* after the 1 in the number.

The number in a negative power of 10 is equal to the number of places the decimal has moved to the *left* from after the 1 in the corresponding number.

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Practice A: Write these as regular numbers without an exponential term. Check your answers at the end of this lesson.

1. 10^{4} = _______________
2. 10^{-4} = ______________
3. 10^{7} = _______________
4. 10^{-5} = ______________

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Numbers in Exponential Notation

Exponential notation is useful in calculations with very large and very small numbers. Though any number can be used as a base, exponential notation most often expresses a value as a number times **10** to a whole-number power.

Examples: 5,250 = 5.25 x 1000 = **5.25×10^{3}**

0.0065 = 6.5 x 0.001 = **6.5×10^{-3}**

Numbers represented in exponential notation have two parts. In **5.25×10^{3}**,

- the ***5.25*** is termed the *significand*, or *mantissa*, or *coefficient*.
- The ***10^{3}*** is the *exponential* term: the *base* and its *exponent* or *power*.

Because *significand* is the standard scientific term, and because *coefficient* and *mantissa* have other meanings in math and chemistry, in these lessons we will refer to the two parts of exponential notation using this terminology:

5.25×10^3
^ ^
significand exponential

You should also learn (and use) any alternate terminology preferred in your course.

Converting Exponential Notation to Numbers

In scientific calculations, it is often necessary to convert from exponential notation to a number without an exponential term. To do so, use these rules.

If the significand is multiplied by a

- *positive* power of 10, move the decimal point in the significand to the *right* by the same number of places as the value of the exponent;

  Examples: $2 \times 10^2 = 2 \times 100 =$ **200**
  ∪∪↑

  $0.0033 \times 10^3 = 0.0033 \times 1{,}000 =$ **3.3**
  ∪∪∪↑

- *negative* power of 10, move the decimal point in the significand to the *left* by the same number of places as the number *after* the minus sign of the exponent.

  Examples: $2 \times 10^{-2} = 2 \times 0.01 =$ **0.02**
  ↑∪∪

  $7{,}653 \times 10^{-3} = 7{,}653 \times 0.001 =$ **7.653**
  ↑∪∪∪

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Practice B: Write these as regular numbers without an exponential term.

1. $3 \times 10^3 =$ ____________________ 2. $5.5 \times 10^{-4} =$ ____________________

3. $0.77 \times 10^6 =$ ____________________ 4. $95 \times 10^{-4} =$ ____________________

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Changing Exponential Notation to Scientific Notation

In chemistry, it is generally required that numbers that are very large or very small be written in **scientific notation**.

Scientific notation, also called *standard* exponential notation, is a subset of exponential notation. Scientific notation represents numeric values using a significand that is *1 or greater, but less than 10*, multiplied by the base 10 to a whole-number power.

This means that to write a number in scientific notation, the *decimal point* in the significand must be moved to *after* the first digit which is not a zero.

Example: 0.050×10^{-2} is written as $\mathbf{5.0} \times 10^{-4}$ in *scientific* notation.

The decimal must be after the first number that is not a zero: the 5.

To convert a number from exponential notation to scientific notation, use these rules.

1. When moving the decimal Y times to make the significand *larger*, make the power of 10 *smaller* by a *count* of Y.

   Example: $0.045 \times 10^{5} = 4.5 \times 10^{3}$
   ∪∪↑

   To convert to scientific notation, the decimal must be after the 4. Move the decimal two times to the right. This makes the significand 100 times *larger*. To keep the same numeric value, lower the power by 2, making the 10^{x} value 100 times smaller.

2. When moving the decimal Y times to make the significand *smaller*, make the power of 10 *larger* by a count of Y.

   Example: $8{,}544 \times 10^{-7} = 8.544 \times 10^{-4}$
   ↑∪∪∪

   To convert to scientific notation, you must move the decimal 3 places to the left. This makes the significand 1,000 times smaller. To keep the same numeric value, increase the exponent by 3, making the 10^{x} value 1,000 times larger.

   Remember, 10^{-4} is 1,000 times *larger* than 10^{-7}.

To learn these rules, it helps to recite each time you move the decimal: "If the number in front gets *larger*, the exponent gets *smaller*. If the number gets smaller, the exponent gets larger."

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Practice C: Change these to scientific notation.

1. $5{,}420 \times 10^{3}$ = ______________
2. 0.0067×10^{-4} = ______________
3. 0.020×10^{3} = ______________
4. 870×10^{-4} = ______________

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Converting Numbers to Scientific Notation

Calculations in exponential notation often use these rules.

- Any number to the zero power equals one.

  $2^{0} = \mathbf{1}$. $42^{0} = \mathbf{1}$. Exponential notation most often uses $\mathbf{10^{0} = 1}$.

- Any number can be multiplied by one without changing its value. This means that any number can be multiplied by 10^{0} without changing its value.

  Example: $42 = 42 \times 1 = 42 \times \mathbf{10^{0}}$ in exponential notation

  $= \mathbf{4.2} \times 10^{1}$ in *scientific* notation.

To convert regular numbers to scientific notation, use these steps.

1. Add **$\times 10^0$** after the number.
2. Apply the rules for scientific notation and moving the decimal.
   - Move the decimal to after the first digit that is not a zero.
   - Adjust the power of 10 to compensate for moving the decimal.

Try a few.

Q. Using those two steps, convert these numbers to scientific notation.

a. 943 b. 0.00036

* * * * * (See *Working Examples* on page 1).

Answers: **943** = 943 x 1 = 943×10^0 = **9.43×10^2** in scientific notation.

0.00036 = 0.00036×10^0 = **3.6×10^{-4}** in scientific notation.

When a number is converted to scientific notation, numbers that are

- larger than one have *positive* exponents (zero and above) in scientific notation;
- smaller than one have *negative* exponents in scientific notation.
- The number of *places* that the decimal moves is the *number* in the exponent.

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Practice D

1. Which lettered parts in Problem 2 below must have negative exponents when written in scientific notation?
2. Change these to scientific notation**.**

a. 6,280 = ______________ b. 0.0093 = _______________

c. 0.741 = _______________ d. 1,280,000 = ________________

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ANSWERS (To make answer pages easy to locate, use a sticky note.)

Pretest. 1. 9.4×10^6 2. 6.7×10^{-5} 3. 4.2×10^4 4. 7.7×10^1

Practice A. 1. 10^4 = 10,000 2. 10^{-4} = 0.0001 3. 10^7 = 10,000,000 4. 10^{-5} = 0.00001

Practice B. 1. 3,000 2. 0.00055 3. 770,000 4. 0.0095

Practice C. 1. 5.42×10^6 2. 6.7×10^{-7} 3. 2.0×10^1 4. 8.7×10^{-2}

Practice D. 1. 2b and 2c. 2a. 6.28×10^3 2b. 9.3×10^{-3} 2c. 7.41×10^{-1} 2d. 1.28×10^6

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Lesson 1B: Calculations Using Exponential Notation

Pretest: If you answer these three questions correctly, you may skip to Lesson 1C. Otherwise, complete Lesson 1B. Answers are at the end of the lesson.

Do *not* use a calculator. Convert final answers to scientific notation.

1. $(2.0 \times 10^{-4})\ (6.0 \times 10^{23})\ =$

2. $\dfrac{10^{23}}{(10^{2})(3.0 \times 10^{-8})} =$

3. $(-6.0 \times 10^{-18}) - (-2.89 \times 10^{-16})\ =$

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Adding and Subtracting Exponential Notation

You need to be able to add and subtract exponential notation without a calculator as a check on your calculator use. Without a calculator, the rule is: exponential notation can be added or subtracted using normal arithmetic *if* you first *convert* all of the numbers to have the *same* exponential term.

Before adding and subtracting, you may convert all the terms to *any* consistent power of 10. However, it often simplifies the arithmetic if you convert all terms to the same exponential as the *largest* of the exponential terms being added or subtracted.

<u>To add and subtract</u> exponential notation, use these steps.

1. Move the decimals so that all of the numbers have the *same* power of 10. Converting to the largest power of 10 is suggested.
2. Add or subtract the significands using standard arithmetic, then add the common power of 10 to the answer.
3. Convert the final answer to scientific notation.

<u>Example:</u>

$$\begin{array}{rcr} 40.7 \times 10^{8} & = & 40.7 \times 10^{8} \\ +\ \underline{22. \ \times 10^{7}} & & +\ \underline{2.2 \times 10^{8}} \\ & & 42.9 \times 10^{8} \end{array} = \boxed{4.29 \times 10^{9}}$$

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Practice A: Try these without a calculator. Convert final answers to scientific notation. After each problem, check your answer at the end of the lesson.

1. $\begin{array}{r} 32.464 \times 10^{1} \\ -\ \underline{16.2 \times 10^{-1}} \end{array}$

2. $(2.25 \times 10^{-6}) + (6.0 \times 10^{-7}) + (21.20 \times 10^{-6})\ =$

3. $(-5.4 \times 10^{-19}) - (-2.18 \times 10^{-18})\ =$

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Multiplying and Dividing Powers of 10

To multiply and divide using exponential notation, handle the exponential terms by these two rules (that must be memorized).

1. When you *multiply* exponentials, you *add* the exponents.

   Examples: $10^3 \times 10^2 = 10^{\mathbf{5}}$ $10^{-5} \times 10^{-2} = 10^{\mathbf{-7}}$ $10^{-3} \times 10^5 = 10^{\mathbf{2}}$

2. When you *divide* exponentials, you *subtract* the exponents**.**

   Examples: $10^3/10^2 = 10^{\mathbf{1}}$ $10^{-5}/10^2 = 10^{\mathbf{-7}}$ $10^{-5}/10^{-2} = 10^{\mathbf{-3}}$

When subtracting, remember: "Minus a minus is a plus." $10^{6-(-3)} = 10^{6+3} = 10^9$

When fractions include several terms, it is often easiest to evaluate the top and bottom separately, then divide.

Example: $\dfrac{10^{-3}}{10^5 \times 10^{-2}} = \dfrac{10^{-3}}{\mathbf{10^3}} = \mathbf{10^{-6}}$

Without using a calculator, simply this fraction to a single exponential term as done in the example above.

$\dfrac{10^{-3} \times 10^{-4}}{10^5 \times 10^{-8}} =$ ______ $=$

* * * * * (See *Working Examples* on page 1).

Answer: $\dfrac{10^{-3} \times 10^{-4}}{10^5 \times 10^{-8}} = \mathbf{\dfrac{10^{-7}}{10^{-3}}} = \mathbf{10^{-7-(-3)}} = \mathbf{10^{-7+3}} = \mathbf{10^{-4}}$

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Practice B: Write answers as 10 to a power. Do *not* use a calculator. Check your answers at the end of the lesson.

1. $10^6 \times 10^2 =$
2. $10^{-5} \times 10^{-6} =$
3. $\dfrac{10^{-5}}{10^{-4}} =$
4. $\dfrac{10^{-3}}{10^5} =$
5. $\dfrac{10^3 \times 10^{-5}}{10^{-2} \times 10^{-4}} =$
6. $\dfrac{10^{-3} \times 10^{23}}{10^{-1} \times 10^{-6}} =$

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Multiplying and Dividing in Exponential Notation

1. When multiplying and dividing using exponential notation, handle the *significands* and *exponents separately*. Do number math using number rules, and exponential math using exponential rules. Then combine the two parts.

   Memorize rule 1, then use it to do the following three problems.

   a. Do not use a calculator: $(2 \times 10^{3})\ (4 \times 10^{23}) =$

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   For numbers, use number rules. 2 times 4 is **8**

   For exponentials, use exponential rules. $10^{3} \times 10^{23} = 10^{\mathbf{26}}$

   Then combine the two parts: $(2 \times 10^{3})\ (4 \times 10^{23}) = \mathbf{8 \times 10^{26}}$

   b. Do the *significand* math on a calculator but the exponential math in your head for $(2.4 \times 10^{-3})\ (3.5 \times 10^{23}) =$

   * * * * *

   Handle significands and exponents separately.

   - Use a calculator for the numbers. 2.4 x 3.5 = **8.4**
   - Do the exponentials in your head. $10^{\mathbf{-3}} \times 10^{\mathbf{23}} = 10^{\mathbf{20}}$
   - Then combine.

   $(2.4 \times 10^{-3})\ (3.5 \times 10^{23}) = (2.4 \times 3.5) \times (10^{-3} \times 10^{23}) = \mathbf{8.4 \times 10^{20}}$

You will learn how much to *round* calculator answers in Module 3. Meanwhile, round numbers and significands in your answers to *two* digits.

   c. Do significands on a calculator but exponentials in your head.

   $$\frac{6.5 \times 10^{23}}{4.1 \times 10^{-8}} = ?$$

   * * * * *

   Answer: $\frac{6.5 \times 10^{23}}{4.1 \times 10^{-8}} = \frac{6.5}{4.1} \times \frac{10^{23}}{10^{-8}} = 1.585 \times [10^{23-(-8)}] = \mathbf{1.6 \times 10^{31}}$

2. When dividing, if an exponential term does *not* have a significand, add a **1 x** in front of the exponential so that the number-number division is clear.

   Try the rule on this problem.

   $$\frac{10^{-14}}{2.0 \times 10^{-8}} =$$

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Answer: $\frac{10^{-14}}{2.0 \times 10^{-8}} = \frac{\mathbf{1}}{2.0} \mathbf{x} \frac{10^{-14}}{10^{-8}} = 0.50 \times 10^{-6} = \mathbf{5.0 \times 10^{-7}}$

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Practice C: Do these in your notebook. *Try* first without a calculator, then check your mental arithmetic with a calculator if needed. Write final answers in scientific notation, rounding significands to two digits. Answers are at the end of the lesson.

1. $(2.0 \times 10^{1})\ (6.0 \times 10^{23}) =$
2. $(5.0 \times 10^{-3})\ (1.5 \times 10^{15}) =$
3. $\frac{3.0 \times 10^{-21}}{2.0 \times 10^{3}} =$
4. $\frac{6.0 \times 10^{-23}}{2.0 \times 10^{-4}} =$
5. $\frac{10^{-14}}{2.0 \times 10^{-3}} =$
6. $\frac{10^{14}}{4.0 \times 10^{-4}} =$

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Practice D

Start by doing every *other* letter. If you get those right, go to the next number. If not, do a few more letters for that set.

1. Convert to scientific notation.

   a. 0.55×10^{5} b. 0.0092×10^{2} c. 940×10^{-6} d. 0.00032×10^{-19}

2. Write these numbers in scientific notation.

   a. 7,700 b. 160,000,000 c. 0.023 d. 0.00067

3. Try these without a calculator. Convert final answers to scientific notation.

   a. $3 \times (6.0 \times 10^{23}) =$
   b. $1/2 \times (6.0 \times 10^{23}) =$
   c. $0.70 \times (6.0 \times 10^{23}) =$
   d. $10^{3} \times (6.0 \times 10^{23}) =$
   e. $10^{-2} \times (6.0 \times 10^{23}) =$
   f. $(0.5 \times 10^{-2})(6.0 \times 10^{23}) =$
   g. $\frac{3.0 \times 10^{24}}{6.0 \times 10^{23}} =$
   h. $\frac{2.0 \times 10^{18}}{6.0 \times 10^{23}} =$
   i. $\frac{1.0 \times 10^{-14}}{4.0 \times 10^{-5}} =$
   j. $\frac{10^{10}}{2.0 \times 10^{-5}} =$

4. Use a calculator for the numbers, but not for the exponents.

   a. $\frac{2.46 \times 10^{19}}{6.0 \times 10^{23}} =$
   b. $\frac{10^{-14}}{7.25 \times 10^{-3}} =$

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ANSWERS

Pretest. In scientific notation: 1. 1.2×10^{20} 2. 3.3×10^{28} 3. 2.83×10^{-16}

Practice A: You may do the arithmetic in any way you choose that results in these answers.

1.
$$\begin{array}{rl} & 32.464 \times 10^{1} \\ - & 16.2 \times 10^{-1} \\ \hline \end{array} = \begin{array}{rl} & 32.464 \times \mathbf{10^{1}} \\ - & 0.162 \times \mathbf{10^{1}} \\ \hline & 32.302 \times \mathbf{10^{1}} \end{array} = \mathbf{3.2302 \times 10^{2}}$$

2.
$$\begin{array}{rl} & 2.25 \times 10^{-6} \\ + & 6.0 \times 10^{-7} \\ + & 21.20 \times 10^{-6} \\ \hline \end{array} = \begin{array}{rl} & 2.25 \times \mathbf{10^{-6}} \\ + & 0.60 \times \mathbf{10^{-6}} \\ + & 21.20 \times \mathbf{10^{-6}} \\ \hline & 24.05 \times \mathbf{10^{-6}} \end{array} = \mathbf{2.405 \times 10^{-5}}$$

3. $(-5.4 \times 10^{-19}) - (-2.18 \times 10^{-18}) = (2.18 \times 10^{-18}) - (5.4 \times 10^{-19}) =$

$$\begin{array}{rl} & 2.18 \times 10^{-18} \\ - & 5.4 \times 10^{-19} \\ \hline \end{array} = \begin{array}{rl} & 2.18 \times 10^{-18} \\ - & 0.54 \times \mathbf{10^{-18}} \\ \hline & \mathbf{1.64 \times 10^{-18}} \end{array}$$

Practice B

1. 10^{8} 2. 10^{-11} 3. 10^{-1} 4. 10^{-8} 5. 10^{4} 6. 10^{27}

Practice C

1. 1.2×10^{25} 2. 7.5×10^{12} 3. 1.5×10^{-24} 4. 3.0×10^{-19}

5. 5.0×10^{-12} 6. 2.5×10^{17}

Practice D

1a. 5.5×10^{4} 1b. 9.2×10^{-1} 1c. 9.4×10^{-4} 1d. 3.2×10^{-23}

2 a. 7.7×10^{3} 2b. 1.6×10^{8} 2c. 2.3×10^{-2} 2d. 6.7×10^{-4}

3a. $3 \times (6.0 \times 10^{23}) = 18 \times 10^{23} = \mathbf{1.8 \times 10^{24}}$ 3b. $1/2 \times (6.0 \times 10^{23}) = \mathbf{3.0 \times 10^{23}}$

3c. $0.70 \times (6.0 \times 10^{23}) = \mathbf{4.2 \times 10^{23}}$ 3d. $10^{3} \times (6.0 \times 10^{23}) = \mathbf{6.0 \times 10^{26}}$

3e. $10^{-2} \times (6.0 \times 10^{23}) = \mathbf{6.0 \times 10^{21}}$ 3f. $(0.5 \times 10^{-2})(6.0 \times 10^{23}) = \mathbf{3.0 \times 10^{21}}$

3g. $\dfrac{3.0 \times 10^{24}}{6.0 \times 10^{23}} = \dfrac{3.0}{6.0} \times \dfrac{10^{24}}{10^{23}} = 0.5 \times 10^{1} = \mathbf{5.0 \times 10^{0}}$ (= **5.0**)

3h. $\dfrac{2.0 \times 10^{18}}{6.0 \times 10^{23}} = 0.33 \times 10^{-5} = \mathbf{3.3 \times 10^{-6}}$ 3i. $\dfrac{1.0 \times 10^{-14}}{4.0 \times 10^{-5}} = 0.25 \times 10^{-9} = \mathbf{2.5 \times 10^{-10}}$

3j. $\dfrac{10^{10}}{2.0 \times 10^{-5}} = \dfrac{1}{2.0} \times \dfrac{10^{10}}{10^{-5}} = 0.50 \times 10^{15} = \mathbf{5.0 \times 10^{14}}$

4a. $\frac{2.46 \times 10^{19}}{6.0 \times 10^{23}} = 0.41 \times 10^{-4} = \mathbf{4.1 \times 10^{-5}}$

4b. $\frac{10^{-14}}{7.25 \times 10^{-3}} = \frac{1.0 \times 10^{-14}}{7.25 \times 10^{-3}} = \frac{1.0}{7.25} \times \frac{10^{-14}}{10^{-3}} = 0.14 \times 10^{-11} = \mathbf{1.4 \times 10^{-12}}$

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Lesson 1C: Tips for Complex Calculations

Pretest: If you get the following problem right and you are in a hurry to get to later lessons, you may skip this lesson. However, this lesson does contain strategies and tips that can speed up your calculations and help when checking your work.

For this problem, use a calculator as needed. Convert your final answer to scientific notation. Check your answer at the end of this lesson.

$$\frac{(3.15 \times 10^{3})(4.0 \times 10^{-24})}{(2.6 \times 10^{-2})(5.5 \times 10^{-5})} =$$

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Choosing a Calculator

If you have not already done so, please read *Choosing a Calculator* under *Notes to the Student* in the preface to these lessons.

Complex Calculations

The prior lessons covered the fundamental rules for calculating with exponential notation. For longer calculations, the rules are the same. The challenges are keeping track of the numbers *and* using the calculator correctly.

The steps below will help you to

- simplify complex calculations,
- minimize data-entry mistakes, and
- quickly *check* your answers.

Let's try the following typical chemistry calculation *two* ways.

$$\frac{(7.4 \times 10^{-2})(6.02 \times 10^{23})}{(2.6 \times 10^{3})(5.5 \times 10^{-5})} =$$

Method 1. Do numbers and exponents separately.

Try the calculation above using the following steps.

a. **Do the numbers on the calculator.** Ignoring the exponentials, use the calculator to multiply all of the *significands* on top. Write the result. Then multiply all the significands on the bottom and write the result. Divide, write your answer rounded to two digits, and then check below.

* * * * * (See *Working Examples* on page 1)

$$\frac{7.4 \times 6.02}{2.6 \times 5.5} = \frac{44.55}{14.3} = \boxed{\mathbf{3.1}}$$

b. **Then exponents.** Starting from the original problem, look only at the powers of 10. Try to solve the exponential math "in your head" with*out* the calculator. Write the answer for the top, then the bottom, then divide.

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$$\frac{10^{-2} \times 10^{23}}{10^{3} \times 10^{-5}} = \frac{10^{21}}{10^{-2}} = 10^{21-(-2)} = \boxed{\mathbf{10^{23}}}$$

c. **Now combine** the significand and exponential and write the final answer.

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3.1 x 10^{23}

Note that by grouping the numbers and exponents *separately*, you did *not* need to enter the *exponents* into your calculator. To multiply and divide powers of 10, you simply add and subtract whole numbers.

Now let's try the calculation a second way.

Method 2. All on the calculator.

Enter *all* of the numbers and exponents into your calculator. (Your calculator manual, which is usually available online, can help.) Solve the top, then the bottom, then divide. Write your final answer in scientific notation. Round the significand to two digits.

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On most calculators, you will need to use an [E *or* EE *or* EXP] key, rather than the *times* key, to enter the power of a "10 to a power" term.

If you needed that hint, try again, and then check below.

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Note how your calculator *displays* the *exponential* term in answers. The exponent may be set apart at the right, sometimes with an **E** in front.

Your calculator answer should be the same as with Method 1: **3.1** x $10^{\mathbf{23}}$.

Which way was easier? "Numbers, then exponents," or "all on the calculator?" How you do the arithmetic is up to you, but "numbers, then exponents" is often quicker and easier.

Using the Reciprocal Key

On a calculator, the **reciprocal key,** [*1/x*] or [x^{-1}] , can save time and steps.

Try the calculation below this way: Multiply the top. Write the answer. Multiply the bottom. Write the answer. Then divide and write your final answer.

$$\frac{74 \times 4.09}{42 \times 6.02} = \underline{\hspace{3cm}} =$$

An alternative to this "top then bottom" method is "bottom, 1/x, top." On the calculator, repeat the above calculation using these steps.

- Multiply the *bottom* numbers first.
- Press the 1/x or x^{-1} button or function on your calculator.
- Then multiply that result by the numbers on top.

Try it.

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You should get the same answer (1.197… **= 1.2**) without having to write the "top-over-bottom" middle step. Your calculator manual can help with using the 1/x function.

The algebra that explains why this works is

$$\frac{A \times B}{C \times D} = \frac{1}{C \times D} \times A \times B = (C \times D)^{-1} \times A \times B$$

The reciprocal key "brings the bottom of a fraction to the top."

Power of 10 Reciprocals

A reciprocal method can be used for powers of 10.

For example, try the following calculation without a calculator. First do the math in your head for the top terms and write the answer. Then evaluate the denominator and write the answer. Divide to get the final answer.

$$\frac{10^{-4} \times 10^{23}}{10^{2} \times 10^{-7}} = \frac{\qquad}{\qquad} =$$

Now try the calculation on paper or "in your head, " but without a calculator, using these steps.

- Multiply the *bottom* terms (by adding the bottom exponents).
- "Bring the bottom exponential to the *top*" by *changing its sign*.
- Multiply that result by the top terms (by adding all of the exponents). Write the answer.

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The steps are *bottom* = 2 + (–7) = **–5** *top* = **+5** – 4 + 23 = 24 *answer* = $10^{\mathbf{24}}$

Why does "bringing an exponent up" change its sign? The algebra is

$$1/10^{x} = (10^{x})^{-1} = 10^{-x}$$

When you take an exponential term to a power, you multiply the exponents.

For *simple* fractions with exponential terms, if your mental arithmetic is good, you should be able to calculate the final answer for the powers of 10 without writing down middle steps. For *longer* calculations, however, writing the "top and bottom" middle-step answers will break the problem into pieces that are easier to manage and check.

Checking Calculator Results

Whenever a complex calculation is done on a calculator, you *must* do the calculation a *second* time to catch errors in calculator use. Calculator results can be checked either by using a different key sequence or by estimating answers.

Below is a method that uses estimation to check multiplication and division in exponential notation. Let's use the calculation that was done in the first section of this lesson as an example.

$$\frac{(7.4 \times 10^{-2})(6.02 \times 10^{23})}{(2.6 \times 10^{3})(5.5 \times 10^{-5})} =$$

Try these steps on the above calculation.

1. ***Estimate* the numbers first.** Ignoring the exponentials, *round* and then multiply all of the top significands, and write the result. *Round* and multiply the bottom significands. Write the result. Then write a *rounded estimate* of the answer when you divide those two numbers, and then check below.

* * * * *

Your rounding might be

$$\frac{7 \times \cancel{6}}{3 \times \cancel{6}} = \frac{7}{3} \approx \mathbf{2}$$ (the ≈ sign means *approximately equals*)

If your mental arithmetic is good, you can estimate the number math on the paper without a calculator. The estimate needs to be fast, but does *not* need to be exact. If needed, evaluate the *rounded* numbers on the calculator.

2. **Evaluate the exponents.** The exponents are simple whole numbers. Try the exponential math without the calculator.

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$$\frac{10^{-2} \times 10^{23}}{10^{3} \times 10^{-5}} = \frac{10^{21}}{10^{-2}} = 10^{21-(-2)} = \mathbf{10^{23}}$$

3. **Combine** the estimated number and exponential answers. Compare this estimated answer to answer found when you did this calculation in the section above using a calculator. Are they close?

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The estimate is 2×10^{23}. The answer with the calculator was 3.1×10^{23}. Allowing for rounding, the two results are close.

If your fast, rounded, "done in your head" answer is *close* to the calculator answer, it is likely that the calculator answer is correct. If the two answers are *far* apart, check your work.

4. **Estimating number division.** If you know your multiplication tables, and if you memorize these simple **decimal equivalents** to help in estimating division, you may be able to do many number estimates without a calculator.

1/2 = 0.50	1/3 = 0.33	1/4 = 0.25	1/5 = 0.20	2/3 = 0.67	3/4 = 0.75

The method used to get your *final* answer should be slow and careful. Your *checking* method should use different steps or calculator keys, and if time is a factor should use rounded numbers.

On timed tests, you may want to do the exact calculation first, and then go back at the end if time is available and use rounded numbers to estimate and check. Your skills at both estimating and finding alternate steps will improve with practice.

When doing a calculation the second time, try not to look back at the first answer. If you look back, by the power of suggestion, you will often arrive the first answer whether it is correct or not.

For complex operations on a calculator, do each calculation a *second* time using rounded numbers and/or different steps or keys.

* * * * *

Practice

In your notebook, do the following calculations.

- First write an *estimate* based on rounded numbers, then exponentials. *Try* to do this estimate without using a calculator.
- Then calculate a more precise answer. You may
  - o plug the entire calculation into the calculator, or
  - o use the "numbers on calculator, exponents on paper" method, or
  - o experiment with both approaches to see which is best for you.

Convert both the estimate and the final answer to *scientific notation*. Round the significand in the answer to two digits.

Use the calculator that you will be allowed to use on quizzes and tests.

To start, try every other problem. If you get those right, go to the next lesson. If you need more practice, do more.

1. $\dfrac{(3.62 \times 10^{4})(6.3 \times 10^{-10})}{(4.2 \times 10^{-4})(9.8 \times 10^{-5})} =$

2. $\dfrac{(1.6 \times 10^{-3})(4.49 \times 10^{-5})}{(2.1 \times 10^{3})(8.2 \times 10^{6})} =$

3. $\dfrac{10^{-2}}{(7.5 \times 10^{2})(2.8 \times 10^{-15})} =$

4. $\dfrac{1}{(4.9 \times 10^{-2})(7.2 \times 10^{-5})} =$

* * * * *

ANSWERS

Pretest: 8.8×10^{-15}

Practice

1. First the estimate. The rounding for the *numbers* might be

$\frac{\cancel{4} \times 6}{\cancel{4} \times 10}$ = **0.6** For the *exponents*: $\frac{10^4 \times 10^{-10}}{10^{-4} \times 10^{-5}} = \frac{10^{-6}}{10^{-9}} = 10^9 \times 10^{-6} = \mathbf{10^3}$

$\approx 0.6 \times 10^3 \approx \mathbf{6 \times 10^2}$ (*estimate*) in scientific notation.

For the *precise* answer, doing numbers and exponents separately,

$\frac{(3.62 \times 10^4)(6.3 \times 10^{-10})}{(4.2 \times 10^{-4})(9.8 \times 10^{-5})} = \frac{3.62 \times 6.3}{4.2 \times 9.8} = \boxed{0.55}$

The exponents are done as in the estimate above.

$= 0.55 \times 10^3 = \boxed{\mathbf{5.5 \times 10^2}}$ (final) in scientific notation.

This is close to the estimate, a check that the more precise answer is correct.

2. You might estimate, for the numbers first,

$\frac{1.6 \times 4.49}{2.1 \times 8.2} = \frac{2 \times 4}{2 \times 8}$ = **0.5** For the exponents: $\frac{10^{-3} \times 10^{-5}}{10^3 \times 10^6} = \frac{10^{-8}}{10^9} = \mathbf{10^{-17}}$

$= 0.5 \times 10^{-17} = \mathbf{5 \times 10^{-18}}$ (estimate)

More precisely, using numbers then exponents, with numbers on the calculator,

$\frac{1.6 \times 4.49}{2.1 \times 8.2} = \boxed{\mathbf{0.42}}$ The exponents are done as in the estimate above.

$0.42 \times 10^{-17} = \boxed{\mathbf{4.2 \times 10^{-18}}}$ This is close to the estimate. Check!

3. Estimate: $\frac{1}{7 \times 3} \approx \frac{1}{20}$ = **0.05** ; $\frac{10^{-2}}{(10^2)(10^{-15})} = 10^{-2-(-13)} = \mathbf{10^{11}}$

$0.05 \times 10^{11} = \mathbf{5 \times 10^9}$ (estimate)

Numbers on calculator: $\frac{1}{7.5 \times 2.8} = \boxed{0.048}$ Exponents – same as in estimate.

FINAL: $\mathbf{0.048} \times 10^{11} = \boxed{\mathbf{4.8 \times 10^9}}$ (close to the estimate)

4. Estimate: $\frac{1}{5 \times 7} \approx \frac{1}{35} \approx$ **0.033** ; $\frac{1}{(10^{-2})(10^{-5})} = 1/(10^{-7}) = \mathbf{10^7}$

$0.033 \times 10^7 \approx \mathbf{3 \times 10^5}$ (estimate)

Numbers on calculator $= \frac{1}{4.9 \times 7.2} = \boxed{0.028}$ Exponents – see estimate.

FINAL: $0.028 \times 10^7 = \boxed{\mathbf{2.8 \times 10^5}}$ (close to the estimate)

* * * * *

SUMMARY –Scientific Notation

1. *Exponential* notation represents numeric values in two parts: a *number* (the significand) times a *base* taken to a *power* (the exponential term).
2. In *scientific* notation, a special case of exponential notation, the signifcand must be a number that is 1 or greater, but less than 10, and the exponential term must be 10 taken to a whole-number power. This puts the decimal point in the significand after the first number which is not a zero.
3. To keep the same numeric value when moving the decimal of a number in base 10 exponential notation, if you
   a. move the decimal *Y times* to make the significand *larger*, make the exponent *smaller* by a *count* of Y;
   b. move the decimal Y times to make the significand smaller, make the exponent larger by a count of Y.
4. To add or subtract exponential notation without a calculator, first convert all of the numbers to the same power of 10, then add or subtract the signifcands, then add the common exponential term to the answer.
5. In calculations using scientific or exponential notation, handle numbers and exponential terms separately. Do numbers by number rules and exponents by exponential rules.
   - When you multiply exponentials, you add the exponents.
   - When you divide exponentials, you subtract the exponents.
6. On complex calculations, it is often easier to do the numbers on the calculator but the exponents on paper.
7. For complex operations on a calculator, do each calculation a *second* time using rounded numbers and/or different steps or keys.

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