

3-1 Study Guide and Intervention

Solving Systems of Equations

Solve Systems Graphically A system of equations is two or more equations with the same variables. You can solve a system of linear equations by using a table or by graphing the equations on the same coordinate plane. If the lines intersect, the solution is that intersection point. The following chart summarizes the possibilities for graphs of two linear equations in two variables.

Graphs of Equations	Slopes of Lines	Classification of System	Number of Solutions
Lines intersect	Different slopes	Consistent and independent	One
Lines coincide (same line)	Same slope, same y-intercept	Consistent and dependent	Infinitely many
Lines are parallel	Same slope, different y-intercepts	Inconsistent	None

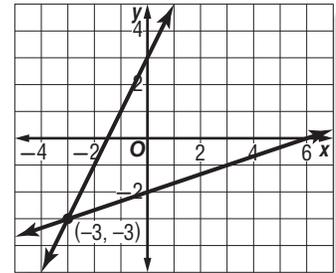
Example Graph the system of equations and describe it as **consistent and independent**, **consistent and dependent**, or **inconsistent**. $x - 3y = 6$
 $2x - y = -3$

Write each equation in slope-intercept form.

$$x - 3y = 6 \rightarrow y = \frac{1}{3}x - 2$$

$$2x - y = -3 \rightarrow y = 2x + 3$$

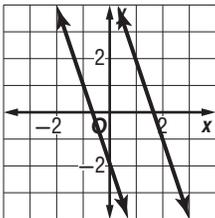
The graphs intersect at $(-3, -3)$. Since there is one solution, the system is consistent and independent.



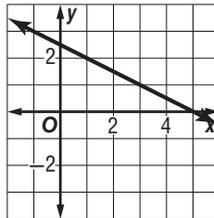
Exercises

Graph each system of equations and describe it as **consistent and independent**, **consistent and dependent**, or **inconsistent**.

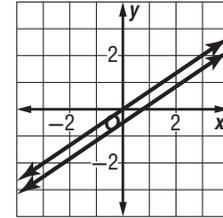
1. $3x + y = -2$
 $6x + 2y = 10$
inconsistent



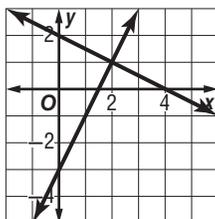
2. $x + 2y = 5$
 $3x - 15 = -6y$ **consistent and dependent**



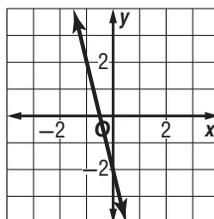
3. $2x - 3y = 0$
 $4x - 6y = 3$
inconsistent



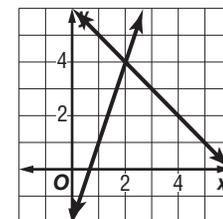
4. $2x - y = 3$
 $x + 2y = 4$ **consistent and independent**



5. $4x + y = -2$
 $2x + \frac{y}{2} = -1$ **consistent and dependent**



6. $3x - y = 2$
 $x + y = 6$ **consistent and independent**



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Solving Systems of Equations

Solve Systems Algebraically To solve a system of linear equations by **substitution**, first solve for one variable in terms of the other in one of the equations. Then substitute this expression into the other equation and simplify. To solve a system of linear equations by **elimination**, add or subtract the equations to eliminate one of the variables.

Example 1 Use substitution to solve the system of equations. $2x - y = 9$
 $x + 3y = -6$

Solve the first equation for y in terms of x .

$$\begin{aligned} 2x - y &= 9 && \text{First equation} \\ -y &= -2x + 9 && \text{Subtract } 2x \text{ from both sides.} \\ y &= 2x - 9 && \text{Multiply both sides by } -1. \end{aligned}$$

Substitute the expression $2x - 9$ for y into the second equation and solve for x .

$$\begin{aligned} x + 3y &= -6 && \text{Second equation} \\ x + 3(2x - 9) &= -6 && \text{Substitute } 2x - 9 \text{ for } y. \\ x + 6x - 27 &= -6 && \text{Distributive Property} \\ 7x - 27 &= -6 && \text{Simplify.} \\ 7x &= 21 && \text{Add } 27 \text{ to each side.} \\ x &= 3 && \text{Divide each side by } 7. \end{aligned}$$

Now, substitute the value 3 for x in either original equation and solve for y .

$$\begin{aligned} 2x - y &= 9 && \text{First equation} \\ 2(3) - y &= 9 && \text{Replace } x \text{ with } 3. \\ 6 - y &= 9 && \text{Simplify.} \\ -y &= 3 && \text{Subtract } 6 \text{ from each side.} \\ y &= -3 && \text{Multiply each side by } -1. \end{aligned}$$

The solution of the system is $(3, -3)$.

Example 2 Use the elimination method to solve the system of equations.

$$\begin{aligned} 3x - 2y &= 4 \\ 5x + 3y &= -25 \end{aligned}$$

Multiply the first equation by 3 and the second equation by 2. Then add the equations to eliminate the y variable.

$$\begin{array}{r} 3x - 2y = 4 \\ 5x + 3y = -25 \end{array} \quad \begin{array}{l} \text{Multiply by } 3 \rightarrow \\ \text{Multiply by } 2 \rightarrow \end{array} \quad \begin{array}{r} 9x - 6y = 12 \\ 10x + 6y = -50 \\ \hline 19x = -38 \\ x = -2 \end{array}$$

Replace x with -2 and solve for y .

$$\begin{aligned} 3x - 2y &= 4 \\ 3(-2) - 2y &= 4 \\ -6 - 2y &= 4 \\ -2y &= 10 \\ y &= -5 \end{aligned}$$

The solution is $(-2, -5)$.

Exercises

Solve each system of equations.

1. $3x + y = 7$
 $4x + 2y = 16$

$(-1, 10)$

4. $2x - y = 7$
 $6x - 3y = 14$

no solution

7. $2x + y = 8$
 $3x + \frac{3}{2}y = 12$

infinitely many

2. $2x + y = 5$
 $3x - 3y = 3$

$(2, 1)$

5. $4x - y = 6$
 $2x - \frac{y}{2} = 4$

no solution

8. $7x + 2y = -1$
 $4x - 3y = -13$

$(-1, 3)$

3. $2x + 3y = -3$
 $x + 2y = 2$

$(-12, 7)$

6. $5x + 2y = 12$
 $-6x - 2y = -14$

$(2, 1)$

9. $3x + 8y = -6$
 $x - y = 9$

$(6, -3)$