Module 5 – Word Problems

Prerequisite: Complete Modules 2 and 4 before starting Module 5.

Timing: Begin Module 5 as soon as you are assigned word-problem calculations.

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Introduction

In this module, you will learn to identify *given* quantities and equalities in word problems. You will then be able to solve nearly all of the initial problems assigned in chemistry with the same conversion-factor method that you used with success in Module 4.

In these lessons, you will be asked to take steps to *organize* your data before you begin to solve a problem. Most students report that by using this method, they then have a better understanding of what steps to take to solve science calculations.

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Lesson 5A: Answer Units -- Single or Ratio?

Types of Units

The units used in measurements can be divided into three types.

- **Single units** have numerators but no denominators. Examples include meters, cubic centimeters, grams, and hours.
- **Ratio units** have one unit in the numerator and one in the denominator. Examples include meters/second and g/mL.
- **Complex units** are all other units, such as 1/sec or (kg·meters^2)/sec^2.

Most of the calculations encountered initially in chemistry involve single units and ratios, but not complex units.

Rules for single units will be covered in this module. The distinctions between single and ratio units will be covered in Module 11. Rules for complex units will be added in Modules 17 and 19.

Rule #1: Know Where You Are Going

In solving science calculations:

To begin each problem, write "WANTED: ? " and the *unit* of the answer.

The first time you read a word problem, look *only* for the *unit* of the answer.

Writing the answer unit first will

- help you choose the correct *given* to start your conversions,
- identify conversions that you will need to solve, and
- tell you when to stop conversions and do the math.

Rules for Answer Units

When writing the WANTED unit, it is important to distinguish between single units and ratio units.

1. An answer unit is a *ratio* unit if a problem asks you to find
   a. "unit X *over one* unit Y," or
   b. "unit X / unit Y," or
   c. "unit X *per* unit Y" where there is no number after *per*.

   Those three expressions are equivalent. All are ways to represent ratio units.

   Example: $\frac{\text{grams}}{\text{mL}}$, also written grams/mL , is a ratio unit.

   If there is no number in the bottom unit, or after the word *per*, the number *one* is understood.

   Example: "Find the speed in miles/hour (or miles per hour)" is equivalent to "find the miles traveled per *one* hour."

   A ratio unit means something per ONE something.

2. An answer unit is a *single* unit if it has a numerator (top term) but no denominator.

   Example: If a problem asks you to find miles, or cm^3, or dollars, it is asking for a single unit.

3. If a problem asks for a "unit per *more than one* other unit," it WANTS a *single* unit.

   Example: If a problem asks for "grams per 100 milliliters," or the "miles traveled in 27 hours," it is asking for a single unit.

   A ratio unit must be something per *one* something.

Writing Answer Units

1. If you WANT a *ratio* unit, write the unit as a *fraction* with a top and a bottom.

   Write: WANTED: **? $\frac{\textbf{miles}}{\textbf{hour}}$ =**

   Do *not* write: WANTED: ? miles/hour" *or* " ? mph

   The slash mark (/), which is read as "per" or "over," is an easy way to type ratios and conversion factors. However, when solving with conversions, *writing* ratio answer units as a *fraction*, with a clear numerator and denominator, is essential in arranging the conversions.

2. If a problem asks for a single unit in the answer, write the WANTED unit in the format

   WANTED: **? miles =** or WANTED: **? mL =**

   Single units have no denominator.

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Practice

Cover the answers below with a sticky note or cover sheet. Then, for each problem, write "WANTED: **?** " and the unit that the problem is asking you to find, using the two rules above. After that WANTED unit, write an equal sign.

Do not finish the problems. Write only the WANTED units.

1. If a car is traveling at 25 miles per hour, how many hours will it take to go 350 miles?
2. If 1.12 liters of a gas at STP has a mass of 3.55 grams, what is the molar mass of the gas, in grams/mole?
3. If a car travels 270 miles in 6 hours, what is its average speed?
4. A student is making 42 signs. She can make 6 signs from one sheet of foamboard. The foamboard costs $18 for 5 sheets. What is the cost of the foamboard for all of the signs?

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ANSWERS

1. The question asks for hours. Write **WANTED**: **? hours =**

This problem is asking for a single unit. *If* the problem asked for hours per one mile, that would be a ratio unit, but hours per *350* miles is asking for a single unit.

2. Write **WANTED**: **? $\frac{\text{grams}}{\text{mole}}$ =** This is a ratio unit. Any unit that is in the form "unit X **/** unit Y" is a ratio unit.

3. In this problem, no unit is specified. However, since the data are in miles and hours, the easiest measure of speed is miles per hour, written

WANTED: ? $\frac{\text{miles}}{\text{hour}}$ = which is a familiar unit of speed. This problem is asking for a ratio unit.

4. **WANTED: ? $ =** *or* **WANTED: ? dollars = .** This problem is asking for the cost of 42 signs, so the answer unit is a single unit. The cost per *one* sign would be a ratio unit.

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Lesson 5B: Mining The DATA

The method we will use *simplify* problems is divide solving into these three parts.

WANTED:

DATA:

SOLVE:

This method of organizing your work will break complex problems into easier pieces. The steps needed to solve will then be more clear.

Rules for DATA

To solve word problems, get rid of the words.

By translating words into numbers, units, and labels, you can solve most initial word problems in chemistry by chaining conversions, as you did in Module 4.

To translate the words, in the DATA section on your paper, write every *number* you encounter as you read the problem. Then add its *unit*, and a *label* that describes what the number and its unit are measuring.

In the initial problems in chemistry, most numbers and units will be parts of equalities, but some will not. It is important to distinguish numbers and units that are parts of equalities from those that are not.

To do so, we need to learn the many ways that equalities can be expressed as words.

Rules for Listing DATA in Word Problems

1. After writing the WANTED unit, on the next line down, write DATA.
2. As you read the problem a second time,
   - Each time you find a number, *stop*. Write the number in the DATA section.
   - After the number, write its *unit* plus a *label* that helps to identify the number.
   - Decide if that number, unit, and label is *paired* with another number, unit, and label as part of an *equality*.
   - Continue until *all* of the numbers in the problem are written in your DATA.
3. In the DATA, *write* each number and unit in the problem as an *equality*

   a. Every time you read *per*. *Per* is written in DATA as an equal sign (=) .

      If a number is shown after *per*, write the number in the equality.

      Example: If you read "$8 per 3 pounds" write in the DATA: "$8 = 3 lb."

      If *no* number is shown after *per*, write *per* as " = **1** "

      Example: If you read "12 sodas per carton," write "12 sodas = 1 carton."

   b. Every time you see a slash mark (/). A slash is the same as *per*.

      Example: If you see "25 miles/hour" write "25 miles = 1 hour"

   c. Every time you see a conversion factor or a ratio unit.

      Write $\frac{\text{25 miles}}{\text{hour}}$ *or* 25 miles/hr under DATA as "25 miles **= 1** hour"

   d. Every time the same quantity is measured using two different units.

      If a problem says, "0.0350 moles of a gas has a volume of 440 mL,"

      write in the DATA: "0.0350 moles of gas = 440 mL"

If a problem says that a bottle is labeled "2 liters (67.6 fluid ounces),"

write in your DATA: "2 liters = 67.6 fluid ounces "

In both of the above are examples, the *same* physical quantity being measured in two different units.

e. Any time two measurements are taken for the same process.

If a problem says, "burning 0.25 grams of candle wax releases 1700 calories of energy," write in your DATA section,

"0.25 grams candle wax = 1700 calories of energy"

Both sides are measures of what happened when this candle burned.

4. Watch for words such as *each* and *every* that mean *one*. *One* is a number, and you want *all* numbers in your DATA table.

   If you read, "Each student was given 2 sodas, " *write* " 1 student = 2 sodas"

5. Note that in writing the WANTED unit, you write "per one" as a ratio unit, and "per more than one" as a single unit.

   In the DATA, however, because *per* is written as an equality, "per one" and "per more than one" can be written in the same way.

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Practice

For each phrase below, write the equality that you will add to your DATA based on the measurements and words. On each side of the equal sign, include a number and a unit. After each unit, if two different entities are being measured in the problem, add additional words that identify what is being measured by the number and unit. After every few, check your answers.

1. The car was traveling at a speed of 55 miles/hour.
2. A bottle of designer water is labeled 0.50 liters (16.9 fluid ounces).
3. Every student was given 19 pages of homework.
4. To melt 36 grams of ice required 2,880 calories of heat.
5. The cost of the three beverages was $5.
6. The molar mass is 18.0 grams H_2O/mole H_2O.
7. Two pencils were given to each student.
8. The dosage of the aspirin is 2.5 mg per kg of body mass.
9. If 125 mL of a gas at STP weighs 0.358 grams, what is the molar mass of the gas?
10. If 0.24 grams of NaOH are dissolved to make 250 mL of solution, what is the concentration of the solution?

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ANSWERS

Terms that are equal may always be written in the reverse order.

If there are two different entities in a problem, attach labels to the units that identify which entity the number and unit are measuring. Doing so will make complex problems much easier to solve.

1. 55 miles = 1 hour (Rule 2b)
2. 0.50 liters = 16.9 fluid ounces (Rule 2d)
3. 1 student = 19 pages (Rule 3)
4. 36 grams ice = 2,880 calories heat (Rule 2e: Equivalent)
5. 3 beverages = $5 (Rule 2e)
6. 18.0 grams H_2O = 1 mole H_2O (Rule 2b)
7. 1 student = 2 pencils (Rule 3)
8. 2.5 mg aspirin = 1 kg of body mass (Rule 2a)
9. 125 mL of gas at STP = 0.358 grams gas (Rule 2d)
10. 0.24 g NaOH = 250 mL of solution (Rule 2e)

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Lesson 5C: Solving For Single Units

DATA Formats If a Single Unit is WANTED

If a problem WANTS a *single* unit, *one* number and unit in the DATA is likely to be

- either a number and its unit that is not paired in an equality with other measurements,
- or a number and its unit that is paired with the WANTED unit in the format

  "? units WANTED = # units *given*"

If a problem WANTS a *single*-unit amount, by the laws of science and algebra, at least *one* item of DATA must be a single-unit amount. In problems that can be solved using conversions, one measurement will be an single unit, and the rest of the DATA will be equalities.

If a single unit is WANTED, *watch* for the one item of data that is a single unit amount. In the DATA, write the single number, unit, and label on a line by itself.

It is a good practice to circle that single unit amount in the DATA, since it will be the *given* number and unit that is used to *start* your conversions.

Variations on the above rules will apply when DATA includes two amounts that are equivalent in a problem. We will address these cases in Module 11. However, for the problems you are *initially* assigned in first-year chemistry, the rules above will most often apply.

To SOLVE

After listing the DATA provided a problem, below the DATA, write SOLVE. Then, *if* you WANT a single unit, write the WANTED and *given* measurements in the format of the *single-unit starting template*.

$$? \text{ unit WANTED} = \# \text{ and UNIT } \textit{given} \bullet \frac{\qquad\qquad}{\text{UNIT } \textit{given}}$$

The template is a "graphic organizer," a way of remembering the rule that your first conversion factor should cancel the *given* ***unit***.

The *given* measurement that is written after the = sign, will be the circled single unit listed in the DATA.

Use the equalities in the DATA (and other fundamental equalities if needed) to convert to the WANTED unit to solve.

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Summary: The 3-Step Method to Simplify Problem Solving

1. WANTED:

When reading a problem for the first time, ask just one question: what will be the unit of the answer? Then, write "WANTED: **?**", the *unit* the problem is asking for, and a *label* that describes what the unit is measuring. Then add an = sign.

Write WANTED ratio units as fractions, and single units as single units.

2. DATA:

Read the problem a second time.

- Every time you encounter a *number*, under DATA, write the number, its unit, and a descriptive label if possible.
- Then see if that number and unit are equal to another number and unit.

If a problem WANTS a single unit, most often *one* measurement will be a single unit and the rest will be equalities. Circle the single unit in the DATA.

3. SOLVE:

If you WANT a single unit, substitute the WANTED and *given* into this format.

$$? \text{ unit WANTED} = \# \text{ and } \textbf{unit } \textit{given} \bullet \frac{\qquad\qquad}{\textbf{unit } \textit{given}}$$

Then, using equalities, convert to the WANTED unit.

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Solve the following problem in your notebook using the 3-step method above, then check your answer.

Q. If a car's speed is 55 miles/hr., how many minutes are needed to travel 85 miles?

* * * * * (the * * * mean cover the answer below, write your answer, then check it.)

Your paper should look like this.

WANTED: ? minutes =

DATA: 55 miles = 1 hour

85 miles

SOLVE: $? \text{ minutes} = 85 \text{ miles} \cdot \frac{1 \text{ hour}}{55 \text{ miles}} \cdot \frac{60 \text{ min.}}{1 \text{ hour}} = \boxed{93 \text{ minutes}}$

You can likely solve *simple* problems without listing WANTED, DATA, SOLVE, but this 3-part method works for *all* problems. It work especially well for the complex problems that soon you will encounter. By using the same three steps for every problem, you will know what to do for all problems. That's the goal.

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Practice

Many science problems are constructed in the following format.

"Equality, equality," then, "**?** WANTED unit = a *given* number and unit."

The problems below are in that format. Using the rules above, solve on these pages or by writing the WANTED, DATA, SOLVE sections in your notebook.

If you get stuck, read part of the answer at the end of this lesson, adjust your work, and try again. Do problems 1 and 3, and problem 2 if you need more practice.

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Problem 1. If 2.2 pounds = 1 kg, what is the mass in grams of 12 pounds?

WANTED: ? (Write the **unit** you are looking for.)

DATA: (Write every number and unit in the problem here. If solving for a single unit, often *one* number and unit is unpaired, and the rest are in equalities, Circle the unpaired single unit.)

SOLVE: (Substitute the above into **?** unit **WANTED** = # and unit ***given*** **•** $\frac{\rule{3em}{0.4pt}}{\text{unit } given}$,

then chain the equalities to find the unit WANTED.)

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<u>Problem 2</u>. If there are 1.6 km/mile, and one mile is 5,280 feet, how many feet are in 0.50 km?

WANTED: ?

DATA:

SOLVE:

?

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<u>Problem 3</u>. If there are 3 floogles per 10 schmoos, 5 floogles/mole, and 3 moles have a mass of 25 gnarfs, how many gnarfs are in 4.2 schmoos?

WANTED:

DATA:

SOLVE:

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ANSWERS

1. WANTED: ? g =

   DATA: 2.2 pounds = 1 kg

   12 pounds

   SOLVE:

   $$? \text{ g} = 12\ \cancel{\text{pounds}} \bullet \frac{1\ \cancel{\text{kg}}}{2.2\ \cancel{\text{pounds}}} \bullet \frac{10^3\text{ g}}{1\ \cancel{\text{kg}}} = \frac{12 \bullet 10^3}{2.2} = 5.5 \times 10^3\text{ g}$$

A single unit is WANTED, and the DATA has one single unit.

Note that the SOLVE step begins with "how many grams equal 12 pounds?"

Fundamental conversions such as kilograms to grams need not be written in your DATA section, but they will often be needed to solve. Be certain that you have mastered the metric system fundamentals.

2. WANTED: ? feet =

   DATA: 1.6 km **=** 1 mile

   1 mile **=** 5,280 feet

   0.50 km

   SOLVE:

   $$? \textbf{ feet} = 0.50\ \cancel{\text{km}} \bullet \frac{1\ \cancel{\text{mile}}}{1.6\ \cancel{\text{km}}} \bullet \frac{5{,}280\textbf{ feet}}{1\ \cancel{\text{mile}}} = \frac{0.50 \bullet 5280}{1.6} = 1{,}600\textbf{ feet}$$

3. WANTED: ? gnarfs =

   DATA: 3 floogles = 10 schmoos

   5 floogles = 1 mole

   3 moles = 25 gnarfs

   4.2 schmoos

   SOLVE:

   First, the SOLVE step restates the question, "how many gnarfs are in 4.2 schmoos?"

   The first conversion is then set up to cancel your *given* unit.

   $$\textbf{? gnarfs} = 4.2\text{ schmoos} \bullet \frac{\rule{4em}{0.4pt}}{\text{schmoos}}$$

   Since only one equality in the DATA contains schmoos, use it to complete the conversion.

   $$\textbf{? gnarfs} = 4.2\ \cancel{\text{schmoos}} \bullet \frac{3\text{ floogles}}{10\ \cancel{\text{schmoos}}}$$

   On the right, you now have floogles. On the left, you WANT gnarfs, so you must get **rid** of floogles. In the next conversion, put floogles where it will **cancel**.

   $$\textbf{? gnarfs} = 4.2\ \cancel{\text{schmoos}} \bullet \frac{3\text{ floogles}}{10\ \cancel{\text{schmoos}}} \bullet \frac{\rule{4em}{0.4pt}}{\text{floogles}}$$

Floogles is in *two* conversion factors in the DATA, but one of them takes us back to schmoos, so let's use the other.

$$\textbf{? gnarfs} = 4.2 \text{ schmoos} \bullet \frac{3 \cancel{\text{floogles}}}{10 \text{ schmoos}} \bullet \frac{1 \text{ mole}}{5 \cancel{\text{floogles}}}$$

Moles must be gotten rid of, but moles has a known relationship with the *answer* unit. Convert from moles to that unit. Since, after unit cancellation, the answer unit is now where you WANT it, stop conversions and do the arithmetic.

$$\textbf{? gnarfs} = 4.2 \text{ schmoos} \bullet \frac{3 \cancel{\text{floogles}}}{10 \text{ schmoos}} \bullet \frac{1 \cancel{\text{mole}}}{5 \cancel{\text{floogles}}} \bullet \frac{25 \textbf{ gnarfs}}{3 \cancel{\text{moles}}} = \frac{4.2 \bullet \cancel{3} \bullet 25}{10 \bullet 5 \bullet \cancel{3}} = \textbf{2.1 gnarfs}$$

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Lesson 5D: Finding the *Given*

Ratio Unit Givens

In chemistry, the initial quantitative topics generally involve solving for single units, so that will be our initial focus as well. Conversion factors may also be used to solve for ratio units, as we did in Lesson 4E.

However, we will defer the most of the rules to use conversions to solve for ratio units until Lesson 11B, when ratio units will be needed to solve for the concentration of chemical solutions. If you need to solve word problems that have ratio-unit answers, now or at any later point, Lesson 11B may be done at any time after completing this lesson.

Single Unit Givens

When solving for single units, the *given* quantity is not always clear.

Example: For the upcoming party, you estimate that each student will consume on average 2.5 cans of soda. If you expect 72 students, the sodas are sold 12 to a carton, and they are on sale at $11 per 3 cartons , what will be the cost of the sodas?

Among all those numbers, which is the *given* needed to begin when you SOLVE?

For a single unit answer, finding the *given* is often a process of elimination. If all of the numbers and units are paired into equalities except one, that one is your *given*.

In your notebook, write the WANTED and DATA sections for the party problem above. (Don't SOLVE yet). Then check your answer below.

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Answer: Your paper should look like this.

WANT: **?** $ = *or* **?** dollars =

DATA: 1 student **=** 2.5 sodas

72 students

12 sodas **=** 1 carton

$ 11 **=** 3 cartons

Since you are looking for a *single* unit, dollars, your data has one number and unit that did not pair up in an equality: 72 students. This is your *given*.

To SOLVE, the rule is

If you WANT a single unit, *start* with a single unit as your *given*.

Apply the above rule, and SOLVE the problem.

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Answer

SOLVE:

If you WANT a single unit, start with the single unit starting template.

$$? \$ = 72 \text{ students} \bullet \frac{\_\_\_\_\_\_}{\text{student}}$$

If you needed that hint, adjust your work and then finish.

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$$\mathbf{? \$ = 72 \text{ students}} \bullet \frac{2.5 \text{ sodas}}{1 \textbf{ student}} \bullet \frac{1 \textbf{ carton}}{12 \text{ sodas}} \bullet \frac{\$ 11}{3 \textbf{ cartons}} = \frac{72 \bullet 2.5 \bullet 11}{12 \bullet 3} = \boxed{\mathbf{\$ 55}}$$

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Practice

For each problem below, use the WANTED, DATA, SOLVE method. If you get stuck, peek at the answers and try again. Do at least two problems. If you plan on taking physics, be sure to do problem 3.

1. A bottle of drinking water is labeled "12 fluid ounces (355 mL)." What is the mass in centigrams of 0.55 fluid ounces of the H_2O? (use the metric *definition* of one gram).

2. You want to mail a large number of newsletters. The cost is 18.5 cents each at bulk rates. On the post office scale, the weight of exactly 12 newsletters is 10.2 ounces. The entire mailing weighs 125 lb. (There are 16 ounces (oz.) in a pound (lb.).)

   a. How many newsletters are you mailing?

   b. What is the cost of the mailing in dollars?

3. If the distance from an antenna on Earth to a geosynchronous communications satellite is 22,300 miles, given that there are 1.61 kilometers per mile, and radio waves travel at the speed of light (3.0×10^8 meters/sec), how many seconds does it take for a signal from the antenna to reach the satellite?

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ANSWERS

1. WANTED: **?** cg =

DATA: 12 fl. oz = 355 mL

0.55 fl. oz

1.00 g $H_2O_{(l)}$ = 1 mL $H_2O_{(l)}$

SOLVE:

$$? \textbf{ cg} = 0.55 \text{ fl. oz.} \cdot \frac{355 \text{ mL}}{12 \text{ fl. oz}} \cdot \frac{1.00 \text{ g } H_2O_{(l)}}{1 \text{ mL } H_2O_{(l)}} \cdot \frac{100 \text{ cg}}{1 \text{ g}} = \mathbf{1{,}600 \text{ cg}}$$

2a. WANTED: **?** newsletters

DATA: 18.5 cents = 1 newsletter

12 exact newsletters = 10.2 ounces

16 oz. = 1 lb. (a definition with infinite sig figs)

125 lb.

SOLVE: $? \text{ newsletters} = 125 \text{ lb.} \cdot \frac{16 \text{ oz.}}{1 \text{ lb.}} \cdot \frac{12 \text{ newsls}}{10.2 \text{ oz.}} = \mathbf{2{,}350 \text{ newsletters}}$

2b. WANTED: **?** dollars

(Strategy: Since you want a single unit, you can start over from your single *given* unit (125 lb.), repeat the conversions above, then add 2 more.

Or you can start from your single unit answer in Part a, and solve using the two additional conversions.

In problems with multiple parts, it is often quicker to use an answer from a previous part to solve for a later part.)

DATA: same as for Part a.

SOLVE: $\textbf{? dollars} = 2{,}350 \text{ newsls} \cdot \frac{18.5 \text{ cents}}{1 \text{ newsl}} \cdot \frac{1 \text{ dollar}}{100 \text{ cents}} = \mathbf{\$\,435}$

3. WANTED: **?** seconds =

DATA: 22,300 miles

1.61 km = 1 mile

3.0×10^8 meters = 1 sec

SOLVE:

$$? \text{ sec} = 22{,}300 \text{ mi.} \cdot \frac{1.61 \text{ km}}{1 \text{ mile}} \cdot \frac{10^3 \text{ meters}}{1 \text{ km}} \cdot \frac{1 \text{ s}}{3.0 \times 10^8 \text{ m}} = \frac{22{,}300 \cdot 1.61 \cdot 10^3}{3.0 \times 10^8} = \mathbf{0.12 \text{ sec}}$$

(This means that the time up *and* back for the signal is 0.24 seconds. You may have noticed this one-quarter second delay during some live broadcasts which bounce video signals off satellites but use faster land-lines for audio, or during overseas phone calls routed through satellites.)

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Lesson 5E: Some Chemistry Practice

Listing Conversions and Equalities

Which is the best way to write DATA pairs —as *equalities*, or in the *fraction* form as conversion-factor ratios? Mathematically, the two forms are identical.

In DATA: the equalities

1.61 km = 1 mile

3.0×10^8 meters = 1 sec.

can be listed as $\frac{1.61 \text{ km}}{1 \text{ mile}}$, $\frac{3.0 \times 10^8 \text{ meters}}{1 \text{ sec.}}$

In these lessons, we will generally write *equalities* in the DATA section. This will emphasize that when solving problems using conversions, you need to focus on relationships between two quantities. However, listing the data in the fraction format is equally valid. Data may be portrayed both ways in textbooks.

Why "Want A Single Unit, Start With A Single Unit?"

Mathematically, the order in which you multiply conversions does not matter. You could solve with your single unit *given* written anywhere on top in your chain of conversions.

However, if you start with a *ratio* as your *given*, when solving for a single unit, there is a 50% chance of starting with a ratio that is inverted. If this happens, the units will never cancel correctly, and you would eventually be forced to start the conversions over. *Starting* with the single unit is a method that automatically arranges your conversions right-side up.

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Practice

Let's do some chemistry.

The problems below supply the DATA needed for conversion factors. In upcoming modules, you will learn how to write these needed conversions automatically even when the problem does not supply them. That small amount of additional information is all that you will need to solve most introductory problems in chemistry.

You're ready. Solve the two problems below in your notebook. Don't let strange terms like *moles* or *STP* bother you. You've done gnarfs. You can do these.

Check your answer after each problem.

1. Water has a molar mass of 18.0 grams H_2O per mole H_2O. How many moles of H_2O are in 450 milligrams of H_2O?

2. If one mole of all gases has a volume of 22.4 liters at STP, and the molar mass of chlorine gas (Cl_2) is 71.0 grams Cl_2 per mole Cl_2 , what is the volume, in liters, of 3.55 grams of Cl_2 gas at STP ?

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ANSWERS

1. WANTED: **? moles H_2O =**

DATA: 18.0 grams H_2O = 1 mole H_2O

450 mg H_2O

SOLVE:

$$\textbf{? moles } \mathbf{H_2O} = 450 \text{ mg } H_2O \cdot \frac{10^{-3}\text{ g}}{1\text{ mg}} \cdot \frac{\textbf{1 mole } \mathbf{H_2O}}{18.0\text{ g } H_2O} = 2.5 \times 10^{-2}\ \textbf{moles } \mathbf{H_2O}$$

Note that we write chemistry data in 3 parts: Number, unit, formula. Writing these complete labels will make complex chemistry problems much easier to solve.

2. WANTED: ? L Cl_2

DATA: 1 mole gas = 22.4 L gas

71.0 g Cl_2 = 1 mole Cl_2

3.55 g Cl_2

SOLVE:

$$\textbf{? L } \mathbf{Cl_2} = 3.55 \text{ g } Cl_2 \cdot \frac{1 \text{ mole } Cl_2}{71.0 \text{ g } Cl_2} \cdot \frac{22.4 \text{ L } Cl_2}{1 \text{ mole } Cl_2} = 1.12\ \textbf{L } \mathbf{Cl_2}$$

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Lesson 5F: Area and Volume Conversions

Timing: Do this lesson *if* you are assigned area and volume conversions based on taking distance conversions to a power, *or if* you are majoring in science or engineering.

Pretest: If you think you know this topic, try the last problem in the lesson. If you can do that problem, you may skip the lesson.

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Area

Area, by definition, is distance squared. All units that measure area can be related to distance units squared. Any equality that relates two *distance* units can be used to calculate an *area* conversion factor. The rule is,

> If an *area* conversion involving two *distance* units is needed, *square* the distance conversion.

For example: Since $\boxed{\text{1 mile = 1.61 km}}$ is a distance conversion,

and any equality can be squared on both sides and still be true,

$(1\text{ mile})^2 = (1.61\text{ km})^2$

$1^2\text{ mile}^2 = (1.61)^2\text{ km}^2$

$\boxed{1\text{ mile}^2 = 2.59\text{ km}^2}$ which can be used as an area conversion.

Based on the above, you can say that "one *square* mile is equal to 2.59 *square* kilometers."

A rule that is important for the math of conversions to work is

> In conversions, write "square units" as **units^2**.

When an area conversion based on a distance conversion is needed, the area conversion can be calculated separately, in steps, as above. However, the area conversion can also be done after the *given* unit as part of the conversions.

The logic is: any two quantities that are equal can be made into a conversion factor written as a fraction. Since the value of any conversion factor **=** 1, and both sides of an equation can be taken to a power and the equation will still be true, then

if $\mathbf{A = B}$, then $\frac{A}{B} = 1$ and $\left(\frac{A}{B}\right)^{\mathbf{2}} = 1^{\mathbf{2}} = 1 = \frac{\mathbf{A^2}}{\mathbf{B^2}}$ which means that

> A conversion factor written as a fraction can be taken to any *power* needed in order to cancel units, and the conversion will remain true (equal to one).

Use that rule to complete this un-finished conversion, then check your answer below.

$$?\text{ miles}^2 = 75\text{ km}^2 \bullet \left(\frac{1\text{ mile}}{1.61\text{ km}}\right)$$

* * * * *

$$? \text{ miles}^2 = 75 \text{ km}^2 \bullet \left(\frac{1 \text{ mile}}{1.61 \text{ km}}\right)^{\mathbf{2}} = 75 \cancel{\text{km}^2} \bullet \frac{1^2 \text{ mile}^2}{(1.61)^2 \cancel{\text{km}^2}} = \frac{75}{2.59} \text{ miles}^2 = \mathbf{29 \text{ miles}^2}$$

To get the km^2 in the *given* to cancel and convert to miles^2 on top, *square* the miles-to-km distance conversion. As above, when you square the conversion, be sure to square everything (each number and each unit) inside the parentheses.

The result above means that the given 75 square kilometers is equal to 29 square miles.

* * * * *

Practice A

1. If 25.4 mm = 1 inch
   a. ? in. = 1.00 mm
   b. ? in^2 = 1.00 mm^2
   c. ? mm^2 = 1.00 in^2
2. A standard sheet of notebook paper has dimensions of 8.5 x 11 inches.
   a. What is the area of one side of the sheet of paper, in square inches?
   b. Using your *part a* answer and the conversion 2.54 cm = 1 inch , calculate the area of one side of the sheet of paper in square centimeters.
3. Under the grid system used to survey the American Midwest, a section, which is one square mile, is 640 acres. The smallest unit of farm land typically surveyed was a "quarter quarter section" of 40 acres. If 1 mile = 1.61 km, 40.0 acres is how many square kilometers?

* * * * *

Volume

Volume, by definition, is distance cubed.

Note that in each of these formulas for the volume of solids, measurements of distance are multiplied *three* times.

- Volume of a rectangular solid = $l \times w \times h$
- Volume of a cylinder = $\pi r^2 h$
- Volume of a sphere = $4/3 \, \pi r^3$

The rules for volume calculations that involve distance units parallel those for area calculations.

- Any unit that measures distance can be used to define a volume unit. The volume unit is simply the distance unit cubed.
- Any equality that relates two distance units can be used as a volume conversion factor by *cubing* the distance conversion.
- In conversions, write "*cubic* units" as **units^3**.

The key rule is:

> If a *volume* conversion that involves two different *distance* units is needed, *cube* the distance conversion.

In chemistry, volume units are used more often than area units. Some key relationships used in distance and volume calculations are

- $\boxed{\text{1 meter = 10 } \textit{deci}\text{meters = 100 } \textit{centi}\text{meters,}}$ which means that
- $\boxed{\text{1 decimeter = 10 centimeters.}}$

Since volume is distance cubed, and one milliliter is *defined* as one cubic centimeter, we can write the metric fundamental rules 4 and 5:

4. $\boxed{1 \text{ cm}^3 = 1 \text{ cc} = 1 \text{ mL}}$ and

5. A cube that is 10 cm x 10 cm x 10 cm = 1 dm x 1 dm x 1 dm =

$\boxed{= 1{,}000 \text{ cm}^3 = 1{,}000 \text{ mL} = 1 \text{ L} = 1 \text{ dm}^3}$ (see Lesson 2A.)

English volume units include cubic inches, fluid ounces, teaspoons, tablespoons, cups, quarts, and gallons.

In these lessons, a direct metric-to-English volume conversion that will be used (and you should memorize) is the "soda can" equality: 12.0 fluid ounces = 355 mL. However, any metric-English distance equality, cubed, also results in a volume conversion.

With the above data, you can perform nearly any type of volume conversion. Try:

Q. Lake Erie is the smallest Great Lake , holding an average 485 km^3 of water. What is this volume in cubic miles? (1.61 km = 1 mile).

* * * * *

WANTED: ? miles^3 (in calculations, write cubic units as units^3.)

DATA: 1.61 km = 1 mile

484 km^3

SOLVE: $? \text{ miles}^3 = 485 \text{ km}^3 \bullet \left(\dfrac{1 \text{ mile}}{1.61 \text{ km}} \right)$

The above conversion is un-finished. Complete it, and then check your answer below.

* * * * *

$$? \text{ miles}^3 = 485 \text{ km}^3 \bullet \left(\frac{1 \text{ mile}}{1.61 \text{ km}} \right)^3 = 485 \text{ km}^3 \bullet \frac{1^3 \text{ mi.}^3}{(1.61)^3 \text{ km}^3} = \frac{485}{4.17} \text{ miles}^3 = \mathbf{116 \text{ miles}^3}$$

To get the *given* km^3 to convert to miles^3, use the miles-to-km distance conversion, cubed. When you cube the conversion, be sure to cube everything inside the parentheses.

To cube 1.61, either multiply 1.61 x 1.61 x 1.61 , *or* use the y^x function on your calculator.

* * * * *

Practice B

Use the conversions above. Do at least every other problem now, but save one or two until just before your test on this material. The more challenging problems are toward the bottom. If you get stuck, read a *part* of the answer, then try again.

1. ? km^3 = 5.00 $miles^3$
2. How many cubic millimeters are in one cubic meter?
3. If 25.4 mm = 1 inch, how many cubic inches are equal to 1.00 cubic millimeters?
4. 0.355 liters
   a. is how many cubic centimeters?
   b. Convert your answer for *part a* to cubic inches.
   c. Using 12 inches = 1 foot, convert your *part b* answer to cubic feet.
5. ? dm^3 = 67.6 fluid ounces (Finish, including the soda-can conversion.)
6. The flathead V-twin engine on the 1947 Indian Chief motorcycle had a 74 cubic inch displacement. What is this displacement in cc's? (1 in. = 2.54 cm)
7. Introduced in 1960, the Chevrolet big block engine, when configured with dual four-barrel carburetors and 11.3:1 compression, developed 425 horsepower at 6200 RPM. The cylinders of this hydrocarbon guzzling behemoth displaced 6.70 L. Immortalized by the Beach Boys, what was this displacement in cubic inches? (1 in. = 2.54 cm)

* * * * *

ANSWERS

Practice A

1\. a. $? \text{ in.} = 1.00 \text{ mm} \cdot \dfrac{1 \text{ inch}}{25.4 \text{ mm}} = 0.0394 \text{ in.}$

b. $? \text{ in}^2 = 1.00 \text{ mm}^2 \cdot \left(\dfrac{1 \text{ inch}}{25.4 \text{ mm}}\right)^{\mathbf{2}} = 1.00\ \cancel{\text{mm}^2} \cdot \dfrac{1^2 \text{ in}^2}{(25.4)^2\ \cancel{\text{mm}^2}} = \dfrac{1}{645} \text{ in}^2 = \mathbf{0.00155\ in^2}$

c. $? \text{ mm}^2 = 1.00 \text{ in}^2 \cdot \left(\dfrac{25.4 \text{ mm}}{1 \text{ in}}\right)^{\mathbf{2}} = 1.00 \text{ in}^2 \cdot \dfrac{(25.4)^2 \text{ mm}^2}{1^2 \text{ in}^2} = \mathbf{645\ mm^2}$

2\. a. Area = length x width = 8.5 in. *x* 11 in. = $\mathbf{93.5\ in^2}$

b. WANTED: **?** cm^2

DATA: 2.54 cm = 1 inch

$\mathbf{93.5\ in^2}$

SOLVE: $? \text{ cm}^2 = 93.5 \text{ in}^2 \cdot \left(\dfrac{2.54 \text{ cm}}{1 \text{ in}}\right)^{\mathbf{2}} = 93.5\ \cancel{\text{in}^2} \cdot \dfrac{(2.54)^2 \text{ cm}^2}{1^2\ \cancel{\text{in}^2}} = \mathbf{603\ cm^2}$

3. WANTED: ? km^2 (in conversions, use exponents for squared, cubed)

   DATA: 1.61 km = 1 mile

   1 section = 1 $mile^2$ = 640 acres (any two equal terms can be used as a conversion)

   40.0 acres

   * * * * *

   SOLVE: $\textbf{?}\ km^2 = 40.0\ acres \cdot \frac{1\ mile^2}{640\ acres} \cdot \left(\frac{1.61\ km}{1\ mile}\right)^{\mathbf{2}} \mathbf{=} \frac{40}{640}\ mi^2 \cdot \frac{2.59\ km^2}{1\ mi^2} = \mathbf{0.162\ km^2}$

Practice B

1. $?\ km^3 = 5.00\ miles^3 \cdot \left(\frac{1.61\ km}{1\ mile}\right)^{\mathbf{3}} \mathbf{=} 5.00\ \cancel{mi^3} \cdot \frac{4.17\ km^3}{1\ \cancel{mi^3}} = \mathbf{20.9\ km^3}$

2. $?\ mm^3 = 1\ meter^3 \cdot \left(\frac{10^3\ mm}{1\ meter}\right)^{\mathbf{3}} \mathbf{=} 1\ meter^3 \cdot \frac{10^9\ mm^3}{1^3\ meter^3} = \mathbf{1 \times 10^9\ mm^3}$

3. $?\ in^3 = 1.00\ mm^3 \cdot \left(\frac{1\ inch}{25.4\ mm}\right)^{\mathbf{3}} \mathbf{=} 1.00\ mm^3 \cdot \frac{1^3\ in^3}{(25.4)^3\ mm^3} = \mathbf{6.10 \times 10^{-5}\ in^3}$

4. a. $?\ cm^3 = 0.355\ L \cdot \frac{1{,}000\ cm^3}{1\ L} = \mathbf{355\ cm^3}$ (metric fundamentals – rule 6)

   b. $?\ in^3 = 355\ cm^3 \cdot \left(\frac{1\ inch}{2.54\ cm}\right)^{\mathbf{3}} \mathbf{=} 355\ cm^3 \cdot \frac{1^3\ in^3}{(2.54)^3\ cm^3} \mathbf{=} \frac{355}{16.4}\ \mathbf{in^3} \mathbf{= 21.7\ in^3}$

   c. $?\ ft^3 = 21.7\ in^3 \cdot \left(\frac{1\ foot}{12\ in}\right)^{\mathbf{3}} \mathbf{=} 21.7\ in^3 \cdot \frac{1^3\ ft^3}{(12)^3\ in^3} \mathbf{=} \frac{21.7}{1{,}728}\ ft^3 \mathbf{= 0.0126\ ft^3}$

5. $?\ dm^3 = 67.6\ fl.\ oz. \cdot \frac{355\ mL}{12.0\ fl\ oz.} \cdot \frac{1\ L}{1{,}000\ mL} \cdot \frac{1\ dm^3}{1\ L} = \mathbf{2.00\ dm^3}$

   (Other conversions may be used if they result in the same answer.)

7. $?\ cm^3 = 74\ in^3 \cdot \left(\frac{2.54\ cm}{1\ in}\right)^{\mathbf{3}} \mathbf{=} 74\ in^3 \cdot \frac{(2.54)^3\ cm^3}{1^3\ in^3} = \mathbf{1{,}200\ cm^3}$

7. WANTED: ? in^3 displacement

   Strategy: This problem has some numbers you don't need. Since you want a displacement in cubic inches, start with the displacement in *liters* as your *given*, then head for the *cm* needed in the metric half of the metric–to-English bridge conversion.

   DATA: 6.70 L displacement

   1 inch = 2.54 cm (metric-English bridge)

   * * * * *

   SOLVE: $\textbf{?}\ in^3 = 6.70\ L \cdot \frac{1{,}000\ cm^3}{1\ L} \cdot \left(\frac{1\ in}{2.54\ cm}\right)^{\mathbf{3}} \mathbf{=} 6{,}700\ cm^3 \cdot \frac{1\ in^3}{(2.54)^3\ cm^3} = \boldsymbol{409}\ \mathbf{in^3}$

* * * * *

Lesson 5G: Density and Solving Equations

Timing: This lesson should be done *if* you are assigned textbook problems on the density of substances that are in the shape of geometric objects such as spheres, cylinders, or rectangular solids.

Pretest: If you think you know this topic, try the last problem in the lesson. If you can do that problem, you may skip the lesson.

* * * * *

Solving Problems Using Mathematical Equations

Calculations in chemistry can generally be solved using conversions, mathematical equations, or both.

Conversions can be used for problems in which all of the relationships can be expressed as two quantities that are equal or equivalent. Equations are required for more complex relationships. In these lessons, when we study gas laws and energy, we will discuss in detail the circumstances in which equations must be used.

Many problems can be solved with either conversions or equations. Conversion methods usually involve less memorization, less algebra, and fewer steps. For most of the early topics in first-year chemistry courses, conversions are the easier way to solve.

An exception is problems involving the density of substances that are in geometric shapes. To calculate volumes, these problems require mathematical equations. (In these lessons, we will call mathematical formulas *equations*, and reserve the term *formula* for chemical formulas.)

Volumes for regular geometric shapes are calculated using equations, including

- Volume of a cube = $(\text{side})^3$
- Volume of a rectangular solid = $l \times w \times h$
- Volume of a cylinder = $\pi r^2 h$
- Volume of a sphere = $4/3\, \pi r^3$

Density is defined as mass per unit of *volume*. In equation form: $\boxed{D = m/V}$.

Density is a ratio: a numeric relationship between mass and volume. Because density is a ratio, it can be used as a conversion factor. Calculations involving density may be solved using either conversions or the density equation.

However, in many density problems equations are required to calculate geometric volumes. If an equation is used for one part, by using the D = m/V equation for the other part, the same equation-solving *method* can be used to solve both parts of the problem.

Both the density equation and the geometric volume equations include *volume* as one of the terms. If we can solve for volume in one equation, we can use that volume to solve for other quantities in the other equation.

In general, if a problem involves *two* equations linked by a common quantity, a useful method to solve is to

- list the equations and DATA for the two equations in separate columns.
- Find the value of the *linked* quantity in the column that does *not* include the WANTED quantity, then
- Add the value of the linked quantity to the other column and solve for the WANTED quantity.

Let us learn this method by example.

Q. If aluminum (Al) has a density of 2.7 g/cm^3, and a 10.80 gram Al cylinder has a diameter of 0.60 cm, what is the height of the cylinder? ($V_{cylinder} = \pi r^2 h$)

Do the following steps in your notebook.

1. First, read the problem and write the *answer* unit. WANTED = ? *unit* and label.
2. To use conversions, at this point we would list the problem's numbers and units, most of them in equalities. However, *if* you see a mathematical *equation* is needed to solve the problem, write that equation in your DATA instead, and draw a $\boxed{\text{box}}$ around it. Then, under the equation, list each *symbol* in the equation, followed by an = sign.
3. If *two* equations are needed to solve the problem, write and $\boxed{\text{box}}$ the two equations in two separate columns. Under each equation, write each symbol in that equation.
4. Usually, one symbol will be the same in both equations. Circle that *linked* symbol in the DATA in both columns. That symbol will have the same *value* in both columns.

Finish those steps and then check your answer below.

* * * * *

At this point, your paper should look like this.

WANTED:	**?** cm height Al cylinder =	
DATA:	$\boxed{V_{cylinder} = \pi r^2 h}$	$\boxed{\textbf{D}\text{ensity} = \textbf{m}\text{ass}/\textbf{V}\text{olume}}$
	V =	D =
	r =	m =
	h =	V =

Next, do the following steps.

5. Write "= **?** WANTED" after the symbol that is WANTED in the problem.
6. Transfer the problem data to the DATA table. After each symbol in the DATA, write the number and unit in the problem that corresponds to that symbol. Use the *units* of the numbers to match up the symbols: grams is mass, mL or cm^3 is volume, etc.
7. After any remaining symbol that does *not* have DATA in the problem, write a **?**.

After you have finished those steps, check your answer below.

* * * * *

Your DATA table should look like this.

DATA:	$\boxed{V_{cylinder} = \pi r^2 h}$	$\boxed{\textbf{D}\text{ensity} = \textbf{m}\text{ass}/\textbf{V}\text{olume}}$
	V = **?**	D = 2.7 g/cm^3
	r = 1/2 diameter = 0.30 cm	m = 10.8 grams
	h = **? WANTED**	V = **?**

8. A fundamental rule of algebra: if you know values for all of the symbols in a mathematical equation except one , you can solve for that missing symbol. If you are missing values for two symbols, you cannot solve for those values directly.

   In the above data, column 1 has two missing values, and column 2 has one. At this point, you can solve for the missing value only in column 2.

   In a problem involving two relationships, usually you will need to solve *first* for the common, linked symbol in the column with*out* the WANTED symbol. Then, use that answer to solve for the WANTED symbol in the other column.

9. When solving an *equation*, solve in symbols before you plug in numbers. In algebra, symbols move faster than numbers with units.

   Solve for the *missing* column 2 data, and then check your answer below.

   * * * * *

   SOLVE: (In column 2, D = m/V ; and we want V. Solve the D equation for V in symbols, then plug in the numbers for those symbols from the DATA.)

   $\boxed{\mathbf{D = m/V}}$

   $$\text{WANTED} = \mathbf{V = \frac{m}{D}} = \frac{10.8\text{ g}}{2.7\text{ g/cm}^3} = 4.0\text{ cm}^3$$

   (In the unit cancellation, 1/(1/X) = X. See Lesson 17C.)

10. Put this solved answer in the DATA. Since the problem is about one specific cylinder, the volume of that cylinder must be the same in both columns. Write your calculated volume in *both* columns.

11. Now solve the equation that contains the WANTED symbol for the WANTED symbol. First solve using the symbols, then plug in the numbers and their units.

    EQUATION: $\boxed{V = \pi r^2 h}$; so

    $$\text{WANTED} = \text{height} = \mathbf{h} = \frac{\mathbf{V}}{\pi \mathbf{r^2}} = \frac{4.0\text{ cm}^3}{\pi\,(0.30\text{ cm})^2} = \frac{4.0\text{ cm}^3}{\pi\,(0.090\text{ cm}^2)} = \boxed{\textbf{14 cm height}}$$

* * * * *

SUMMARY: Steps for Solving With Equations

1. First write WANTED = ? and the unit you are looking for.
2. When you see that you need a mathematical equation to solve, under DATA, write the equation.
3. If you need two equations, write them in separate columns.
4. Under each equation, list each symbol in that equation.
5. Write "**?** WANTED" after the WANTED symbol in the problem.
6. After each symbol, write numbers and units from the problem. Use the units to match the numbers and units with the appropriate symbol.
7. Label remaining symbols without DATA with a **?**
8. Circle symbols for variables that are the same in both equations.
9. Solve equations in symbols before plugging in numbers.
10. Solve for ? in the column with *one ?* first.
11. Write that answer in the DATA for both columns, then solve for the WANTED symbol.

With practice, these steps will become automatic.

* * * * *

Flashcards

Using the table below, cover the answer column, then put a check by the questions in the left column you can answer quickly and automatically. For the others, make flashcards using the method in Lesson 2D.

One-way cards (with notch at top right): Back Side -- Answers

One-way cards (with notch at top right):	Back Side -- Answers
Density =	Mass/Volume
Volume of a cube =	$(\text{side})^3$
Volume of a sphere =	$4/3\ \pi\ r^3$
Volume of a cylinder =	$\pi\ r^2\ h$

* * * * *

Practice: Practice any needed flashcards above, then try two of the problems below. Save one problem for your next study session. Add the flashcards to your stack for weekly then monthly review.

Use the steps for solving with equations above. Answers are at the end of this lesson. If you get stuck, read a part of the answer, and then try again.

1. If the density of lead is 11.3 grams per cubic centimeter, what is the mass of a ball of lead that is 9.0 cm in diameter?
2. A gold American Eagle $50 coin has a diameter of 3.26 cm and mass of 36.7 grams. Assuming that the coin is in the approximate shape of a cylinder and is made of gold

alloy (density = 15.5 g/mL), what is the height of the cylinder (the thickness of the coin)?

3. If sugar cubes with the length on a side of 1.80 cm have an average mass of 6.30 grams, what is the density of the sugar cubes, in grams per cubic centimeter?

* * * * *

ANSWERS

1 . **WANTED**: ? grams lead

DATA:

$\boxed{V_{sphere} = 4/3\pi r^3}$ $\boxed{\textbf{D}\text{ensity} = \textbf{m}\text{ass}/\textbf{V}\text{olume}}$

V = **?** D = 11.3 g/cm^3

r = 1/2 diameter = 4.5 cm m = **? WANTED**

V = **?**

Strategy: First solve for the ? in the column with **one ?**. Then use that answer to solve for the variable that is WANTED in the other column.

SOLVE: Column 1 has one **?**, and column 2 has two. Solve column one first.

$$\textbf{?} = \textbf{V} = 4/3\ \pi\ r^3 = 4/3\ \pi\ (4.5\text{ cm})^3 = \mathbf{38\textit{2}\ cm^3}$$

In problems that solve in steps, carry an extra sig fig until the final step.

Add this answer to the *volume* DATA in *both* columns. Then solve the Column 2 equation for the WANTED mass. First solve in symbols, then plug in the numbers.

If needed, adjust your work, then finish.

* * * * *

$\boxed{D = \mathbf{m}/V}$ and **m**ass is WANTED,

$$\text{WANTED} \mathbf{=} \mathbf{m} = D \bullet V = 11.3\ \frac{g}{cm^3} \bullet 382\ cm^3 = \mathbf{4.3 \times 10^3\ grams}\ \text{(2 sig figs)}$$

Units must be included and must cancel to give the WANTED unit.

Use the sig figs in the original data to determine the sig figs in the final answer.

You can also solve the column 2 data for grams using conversion factors.

$$?\ g = 382\ cm^3 \bullet \frac{11.3\ g}{1\ cm^3} = 4.3 \times 10^3\ g$$

2. (Hint: You will need 1 mL = 1 cm^3)

WANTED: **? cm** height of gold cylinder (thickness of coin)

DATA:

$\boxed{V_{cylinder} = \pi r^2 h}$ $\boxed{\text{D} = \textbf{m}\text{ass}/\textbf{V}\text{olume}}$

V = **?** D = 15.5 g/mL

r = 1/2 diameter = 1.63 cm m = 36.7 grams

h = **? WANTED** V = **?**

Strategy: First complete the column with one **?**, then use that answer to solve for the variable WANTED in the other column. Column 1 has two **?** and column 2 has one.

SOLVE: $\boxed{D = m/V}$;

WANTED = V = $\frac{\mathbf{m}}{\mathbf{D}} = \frac{36.7\ g}{15.5\ g/mL}$ = **2.36*8* mL** (Carry extra sig fig until end)

(For help with the unit cancellation in equations, see Lesson 17C.)

Fill in that **V**olume in both columns. Then solve the equation that contains the WANTED symbol, first in symbols, and then with numbers.

EQUATION: $\boxed{V = \pi r^2 \mathbf{h}}$

$$\textbf{WANTED} = \text{height} = \mathbf{h} = \frac{\mathbf{V}}{\pi\,\mathbf{r^2}} = \frac{2.36\mathit{8}\ mL}{\pi\,(1.63\ cm)^2} = \frac{2.368\ \mathbf{cm^3}}{8.347\ \mathbf{cm^2}} = \boxed{\mathbf{0.284\ cm}}$$

Note carefully the unit cancellation above. By changing mL to cm^3 (they are identical), the units are *consistent*. They then cancel properly.

A *height* of a cylinder, or *thickness* of a coin, must be in *distance* units such as cm.

Your work must include unit cancellation, and your answers must include correct units to be correct.

3. WANTED: **?** $\frac{grams}{cm^3}$ sugar cube =

DATA: 6.30 grams sugar

Side of cube = 1.80 cm

Strategy: This one is tricky because you are not told that you need to calculate volume. Note, however, that you WANT grams per *cubic* cm. You are given grams and cm. In density problems, be on the lookout for a volume calculation.

The equation for the volume of a cube is $V_{cube} = (side)^3$.

If you needed that hint, adjust your work and try the question again.

* * * * *

DATA: $\boxed{V_{cube} = (Side)^3}$ $\boxed{D = \mathbf{m}ass/\mathbf{V}olume}$

V = **?**	D = **? WANTED**
side = 1.80 cm	m = 6.30 grams sugar
	V = **?**

SOLVE: First solve the column with *one* **?** and put that answer in both columns.

Volume of cube = $(side)^3 = (1.80\ cm)^3 = \boxed{5.832\ \mathbf{cm^3}}$

Now solve for the WANTED symbol in the other equation. (In this problem, the density equation is already written in a form that solves for the WANTED symbol.) Then substitute the numbers and units and solve.

$$D = \textbf{? WANTED} = \frac{mass}{volume} = \frac{6.30\ g}{5.832\ cm^3} = 1.08\ \frac{g}{cm^3}$$

* * * * *

Summary: Word Problems

1. Organize your work into 3 parts: WANTED, DATA, and SOLVE.
2. First, under WANTED, write the unit you are looking for. Add a label that describes what the unit is measuring.
3. If a *ratio* unit is WANTED, write the unit as a fraction with a top and a bottom.
4. Under DATA, to solve with conversions,
   - write every number in the problem. Attach the units to the numbers. If the problem involves more than one substance, add a label to the unit and number that identifies which substance is being measured.
   - If numbers and units are paired with other numbers and units, write the DATA as equalities.
   - Write *per* or a slash (**/**) in the data as **=** . If no number is given after the *per* or /, write *per* or **/** as **= 1** .
   - Write as equalities two different measurements of the same entity, or any units and labels that are equivalent or mathematically related in the problem.
5. To SOLVE, if you WANT a single unit, start with a single number and unit as your *given* and chain the conversions.
6. For problems that require mathematical equations to solve,
   - write the equations in your DATA.
   - List each symbol in the equation below the equation.
   - Match the data in the problem to the symbols.
   - Solve in symbols before plugging in numbers.
7. For problems requiring two equations to solve, solve the two equations separately. Solve for the linked variable in the non-WANTED column first. Use that answer as DATA to solve for the WANTED symbol in the other column.

* * * * *

# # # # #