DERIVATIVE RULES OF TRANSCENDENTAL FUNCTIONS WITH THE CHAIN RULE

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Assume that the appropriate criteria have been meet for $f(u(x))$, let $u' = \frac{du}{dx}$ and recall that the chain rule is

$$\frac{d}{dx}(f(u)) = f'(u) \cdot u'$$

and the chain rule combined with the power rule is

$$\frac{d}{dx}(u^n) = nu^{n-1} \cdot u'.$$

1. Exponential & Logarithmic Functions

(1) $\frac{d}{dx}(e^u) = u'e^u$

(2) $\frac{d}{dx}(a^u) = u'a^u \ln a$

(3) $\frac{d}{dx}(\ln|u|) = \frac{u'}{u}$

(4) $\frac{d}{dx}(\log_a u) = \frac{u'}{u \ln a}$

2. Trigonometric Functions

(1) $\frac{d}{dx}(\sin u) = u' \cos u$

(2) $\frac{d}{dx}(\cos u) = -u' \sin u$

(3) $\frac{d}{dx}(\tan u) = u' \sec^2 u$

(4) $\frac{d}{dx}(\csc u) = -u' \csc u \cot u$

(5) $\frac{d}{dx}(\sec u) = u' \sec u \tan u$

(6) $\frac{d}{dx}(\cot u) = -u' \csc^2 u$

3. Inverse Trigonometric Functions

(1) $\frac{d}{dx}\left(\sin^{-1} u\right) = \frac{u'}{\sqrt{1-u^2}}$

(2) $\frac{d}{dx}\left(\cos^{-1} u\right) = -\frac{u'}{\sqrt{1-u^2}}$

(3) $\frac{d}{dx}\left(\tan^{-1} u\right) = \frac{u'}{1+u^2}$

(4) $\frac{d}{dx}\left(\csc^{-1} u\right) = -\frac{u'}{u\sqrt{u^2-1}}$

(5) $\frac{d}{dx}\left(\sec^{-1} u\right) = \frac{u'}{u\sqrt{u^2-1}}$

(6) $\frac{d}{dx}\left(\cot^{-1} u\right) = -\frac{u'}{1+u^2}$

4. Hyperbolic Functions

(1) $\frac{d}{dx}(\text{sinh}u) = u'\text{cosh}u$

(2) $\frac{d}{dx}(\text{cosh}u) = u'\text{sinh}u$

(3) $\frac{d}{dx}(\text{tanh}u) = u'\text{sech}^2u$

(4) $\frac{d}{dx}(\text{csch}u) = -u'\text{csch}u\text{coth}u$

(5) $\frac{d}{dx}(\text{sech}u) = -u'\text{sech}u\text{tanh}u$

(6) $\frac{d}{dx}(\text{coth}u) = -u'\text{csch}^2u$

5. Inverse Hyperbolic Functions

(1) $\frac{d}{dx}\left(\sinh^{-1}u\right) = \frac{u'}{\sqrt{1+u^2}}$

(2) $\frac{d}{dx}\left(\cosh^{-1}u\right) = \frac{u'}{\sqrt{u^2-1}}$

(3) $\frac{d}{dx}\left(\tanh^{-1}u\right) = \frac{u'}{1-u^2}$

(4) $\frac{d}{dx}\left(\text{csch}^{-1}u\right) = -\frac{u'}{|u|\sqrt{u^2+1}}$

(5) $\frac{d}{dx}\left(\text{sech}^{-1}u\right) = -\frac{u'}{u\sqrt{1-u^2}}$

(6) $\frac{d}{dx}\left(\coth^{-1}u\right) = \frac{u'}{1-u^2}$